Contents

Chapter 1

The Physics Case for BTeV

1.1 Introduction

Experimental particle physics seeks answers to many questions about nature. Some central issues include:

- How are fermion masses generated?
- Why is there a family structure?
- Why are there three families rather than one?

The Standard Model [1] describes current experimental data quite well, but does not directly address these questions. Thus far all predictions are consistent with experiment. Symmetries and symmetry violations are crucially important physics phenomena. Weak decays are known to violate parity, P, and the product of charge-conjugation and parity, CP in the K^o and B^o systems. [2]. That the three family structure allows CP violation to occur naturally via quark mixing is an important clue that we are on the right track. However, the Standard Model is more of a description than an explanation.

The magnitude of CP violation is intimately tied to the question of "baryogenesis," or how did the Universe get rid of the anti-baryons? A possible solution was first proposed by Sakharov [3]. It requires three ingredients: CP violation, lack of thermal equilibrium at some time and baryon non-conservation. The Standard Model provides the third component via quantum corrections to anomaly diagrams. Inflation can provide the lack of thermal equilibrium. Although the Standard Model incorporates CP violation, it is believed that the amount is far too small. Of course we may find that the Standard Model explanation is incorrect.

We describe here a program of measurements that need to be performed in order to test whether the Standard Model indeed describes quark mixing and CP violation. There are many important experimental measurements to be made. We will describe the reasons why these measurements are crucial. We will also point out the important tests that probe physics beyond the Standard Model.

There are many other interesting and important physics topics concerning issues of heavy quark production, the phenomenology of weak decays, CPT violation, etc., that we do not discuss here. It should be kept in mind that other areas of interesting physics can be addressed by BTeV.

1.2 The CKM Matrix

1.2.1 Introduction

The physical point-like states of nature that have both strong and electroweak interactions, the quarks, are mixtures of base states described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [4],

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{1.1}$$

The unprimed states are the mass eigenstates, while the primed states denote the weak eigenstates. The V_{ij}'s are complex numbers that can be represented by four independent real quantities. These numbers are fundamental constants of nature that need to be determined from experiment, like any other fundamental constant such as α or G. In the Wolfenstein approximation the matrix is written as [5]

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta(1-\lambda^2/2)) \\ -\lambda & 1-\lambda^2/2 - i\eta A^2\lambda^4 & A\lambda^2(1+i\eta\lambda^2) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{1.2}$$

This expression is accurate to order λ^3 in the real part and λ^5 in the imaginary part. It is necessary to express the matrix to this order to have a complete formulation of the physics we wish to pursue. The constants λ and A have been measured using semileptonic s and b decays [6]; $\lambda \approx 0.22$ and $A \approx 0.8$. The phase η allows for CP violation. There are experimental constraints on ρ and η that will be discussed below.

1.2.2 Unitarity Triangles

The unitarity of the CKM matrix[1] allows us to construct six relationships. These equations may be thought of as triangles in the complex plane. They are shown in Fig. 1.1

In the **bd** triangle, the one usually considered, the angles are all thought to be relatively large. It is described by:

$$V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0 \quad . \tag{1.3}$$

To a good approximation

$$|V_{ud}^*| \approx |V_{tb}| \approx 1, \tag{1.4}$$

[1]Unitarity implies that any pair of rows or columns are orthogonal.

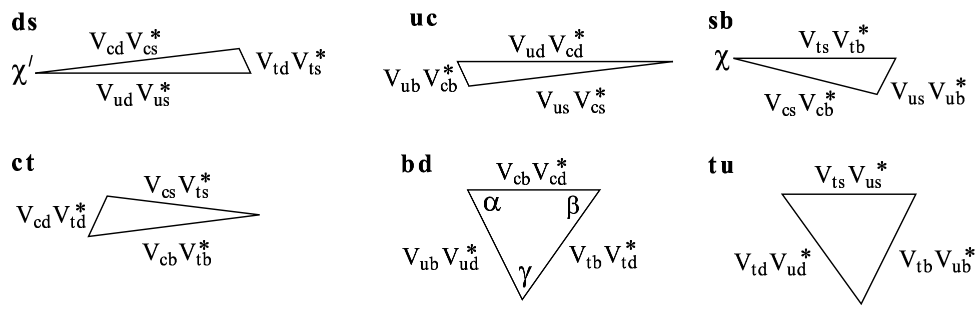

Figure 1.1: The six CKM triangles. The bold labels, i.e **ds** refer to the rows or columns used in the unitarity relationship. (Angles are also shown for later reference.)

which implies

$$\frac{V_{ub}}{V_{cb}} + \frac{V_{td}^*}{V_{cb}} + V_{cd}^* = 0 \quad . \tag{1.5}$$

Since $V_{cd}^* = \lambda$, we can define a triangle with sides

$$\begin{aligned} & 1 && (1.6) \\ \left|\frac{V_{td}}{A\lambda^3}\right| &= \sqrt{(\rho-1)^2+\eta^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right| && (1.7) \\ \left|\frac{V_{ub}}{A\lambda^3}\right| &= \sqrt{\rho^2+\eta^2} = \frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|. && (1.8) \end{aligned}$$

This CKM triangle is depicted in Fig. 1.2.

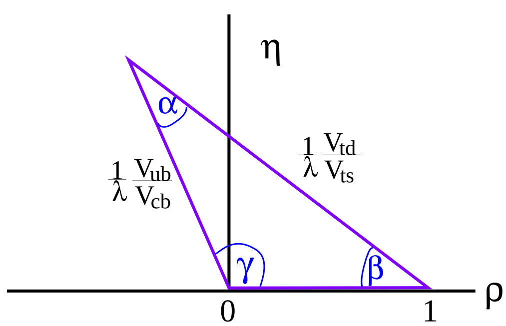

Figure 1.2: The CKM triangle shown in the $\rho - \eta$ plane. The left side is determined by $|V_{ub}/V_{cb}|$ and the right side can be determined using mixing in the neutral B system. The angles can be directly determined by making measurements of CP violation in B decays.

We know two sides already: the base is defined as unity and the left side is determined within a relatively large error by the measurements of $|V_{ub}/V_{cb}|$ [7]. The right side can, in principle, be determined using mixing measurements in the neutral B system. However, even though B_d mixing is well measured, there are theoretical parameters needed that have large uncertainties (see the next section). Later we will discuss other measurements that can determine this side, especially that of B_s mixing. The figure also shows the angles α, β, and γ. These angles can be determined by measuring CP violation in the B system.

Aleksan, Kayser and London [8] created an alternative parameterization expressing the CKM matrix in terms of four independent phases. These are taken as:

$$\beta = arg\left(-\frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*}\right), \qquad \gamma = arg\left(-\frac{V_{ub}^*V_{ud}}{V_{cb}^*V_{cd}}\right),$$
$$\chi = arg\left(-\frac{V_{cs}^*V_{cb}}{V_{ts}^*V_{tb}}\right), \qquad \chi' = arg\left(-\frac{V_{ud}^*V_{us}}{V_{cd}^*V_{cs}}\right) \ . \tag{1.9}$$

These angles are shown in Fig. 1.1; we have changed the confusing notation of Aleksan *et al.* from ϵ, ϵ' to χ and χ'. We will address the usefulness of this parameterization in section 1.12.2.

1.2.3 Neutral B Mixing

Neutral B mesons can transform to their anti-particles before they decay. The Standard Model diagrams for B_d mixing are shown in Fig. 1.3. (The diagrams for B_s mixing are similar with s quarks replacing d quarks.) Although u, c and t quark exchanges are all shown, the t quark plays a dominant role, mainly due to its mass, since the amplitude of this process grows with the mass of the exchanged fermion.

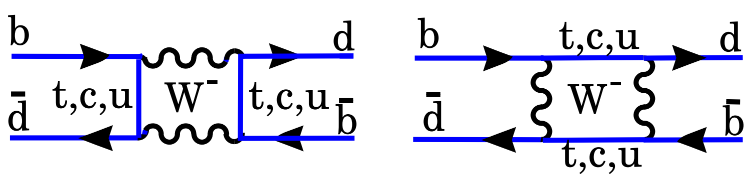

Figure 1.3: The two diagrams for B_d mixing.

The probability of B^o mixing is given by [9]

$$r = \frac{N\left(\overline{B}^o\right)}{N\left(B^o\right)} = \frac{x^2}{2+x^2} \ , \ \text{where} \tag{1.10}$$

$$x \equiv \frac{\Delta m}{\Gamma} = \frac{G_F^2}{6\pi^2}B_B f_B^2 m_B \tau_B |V_{tb}^* V_{td}|^2 m_t^2 F\left(\frac{m_t^2}{M_W^2}\right)\eta_{QCD}, \tag{1.11}$$

where $B_B f_B^2$ is related to the probability of the d and $\bar{b}$ quarks forming a hadron and must be estimated theoretically; F is a known function given by Inami and Lin [10] that increases approximately as m_t^2, and η_{QCD} is a QCD correction, with a value ≈ 0.8 [11]. By far the largest uncertainty arises from the unknown decay constant, f_B. This number is associated with the coupling between the B and the W^-. The product $f_B|V_{ub}|$ could in principle be determined by finding the decay rate of $B^+ \to \mu^+\nu$ or $B^+ \to \tau^+\nu$, both of which are very difficult to measure. Since

$$|V_{tb}^* V_{td}|^2 \propto |(1-\rho-i\eta)|^2 = (\rho-1)^2+\eta^2, \tag{1.12}$$

measuring mixing gives a circle centered at (1,0) in the $\rho - \eta$ plane. The best recent mixing measurements have come from a variety of sources [12], yielding a value (for B_d) of $\Delta m = (0.472 \pm 0.017) \times 10^{12}\, \hbar s^{-1}$.

The right-hand side of the triangle can be determined by measuring B_s mixing using the ratio

$$\frac{\Delta m_s}{\Delta m_d} = \left(\frac{B_s}{B}\right)\left(\frac{f_{B_s}}{f_B}\right)^2\left(\frac{m_{B_s}}{m_B}\right)\left|\frac{V_{ts}}{V_{td}}\right|^2, \tag{1.13}$$

where

$$\left|\frac{V_{td}}{V_{ts}}\right|^2 = \lambda^2\left[(\rho-1)^2+\eta^2\right]. \tag{1.14}$$

The uncertainty in using the B_d mixing measurement to constrain ρ and η is largely removed since many sources of theoretical uncertainty cancel in the ratio of the first two factors in equation (1.13), which is believed to be known to ±20% [13].

1.2.4 Current Status of the CKM Matrix

Since λ and A have been measured, λ precisely, and A to about ±7% [14], we can view other measurements as giving constraints in the $\rho - \eta$ plane. We will leave the inclusion of CP violation measurements in B^o decay to a later section. One constraint on ρ and η is given by the K_L^o CP violation measurement (ϵ) [15] :

$$\eta\left[(1-\rho)A^2(1.4\pm0.2)+0.35\right]A^2\frac{B_K}{0.75} = (0.30\pm0.06), \tag{1.15}$$

where B_K is parameter that cannot be measured and thus must be calculated. A reasonable range is $0.9 > B_k > 0.6$, given by an assortment of theoretical calculations [15]; this number is one of the largest sources of uncertainty. Other constraints come from current measurements on V_{ub}/V_{cb}, B_d mixing and a lower limit on B_s mixing. Measurements of $|V_{ub}/V_{cb}|^2$ are proportional to $\rho^2+\eta^2$ and thus form a circular constraint in the $\rho - \eta$ plane centered at (0,0). Similarly, mixing measurements form a circular constraint centered on (1,0). The current status of constraints on ρ and η is shown in Figure 1.4 from Hocker *et al.* [16]. The confidence level contours are generated using a method where theoretical parameters, such as f_B and B_K, are given equal probability to exist within arbitary selected limits. We caution the reader that this plot is only a guide, since the measured quantities all have large or even dominant errors due to theoretical models. This analysis is in good agreement with that of Rosner [17] and Plaszczynski and Schune [18], but not in agreement with Ciuchini *et al.* [19], who extract what we view as unreasonably small errors from the data [20].

Recent measurements of ϵ'/ϵ in $K_L \to \pi\pi$ decay determine η directly [2]. However, the theoretical errors are so large that all that can be said is that the measurement is consistent with the allowed region.

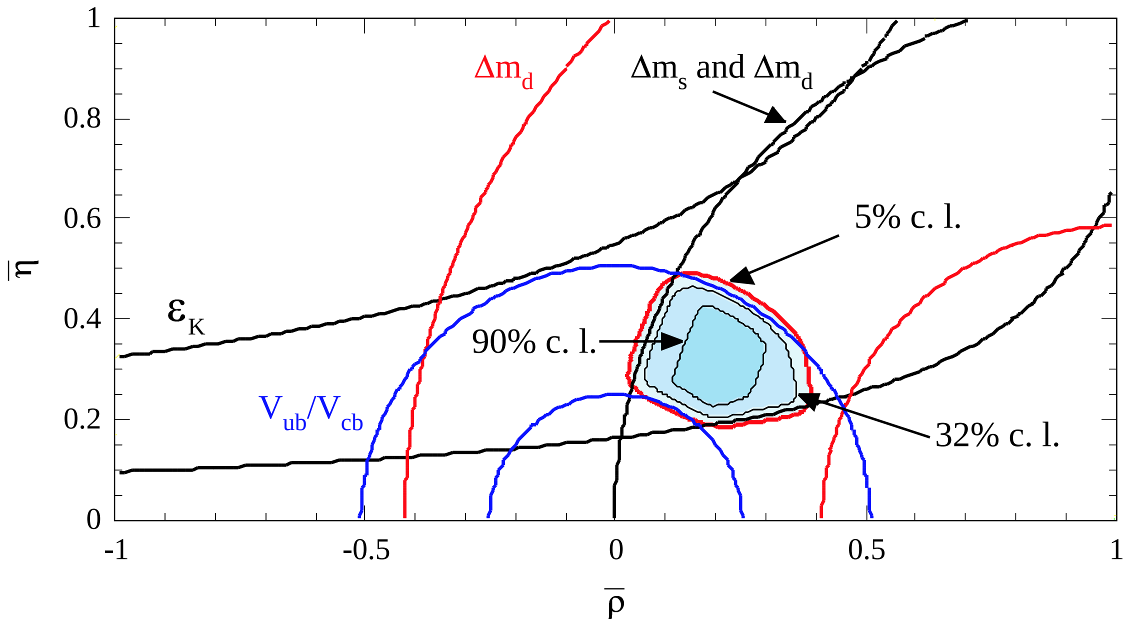

Figure 1.4: The regions in $\overline{\rho}-\overline{\eta}$ space (shaded), where $\overline{\rho}=\rho(1-\lambda^2/2)$ and $\overline{\eta}=\eta(1-\lambda^2/2)$, consistent with measurements of CP violation in K_L^o decay (ϵ), V_{ub}/V_{cb} in semileptonic B decay, B_d^o mixing, and the excluded region from limits on B_s^o mixing. The allowed region is defined by a fit from Hocker *et al.* [16]. The large width of the B_d mixing band is dominated by the uncertainty in $B_B f_B^2$. The lines that are not specified are at 5% confidence level.

1.3 CP Violation in Charged B Decays

The fact that the CKM matrix is complex allows CP violation. The theoretical basis of the study of CP violation in B decays was given in a series of papers by Carter and Sanda, and Bigi and Sanda [21]. We start with charged B decays. Consider the final states $f^{\pm}$ which can be reached by two distinct weak processes with amplitudes $\mathcal{A}$ and $\mathcal{B}$, respectively.

$$\mathcal{A}=a_s e^{i\theta_s} a_w e^{i\theta_w}, \quad \mathcal{B}=b_s e^{i\delta_s} b_w e^{i\delta_w} \quad . \tag{1.16}$$

The strong phases are denoted by the subscript s and weak phases are denoted by the subscript w. Under the CP operation the strong phases are invariant but the weak phases change sign, so

$$\overline{\mathcal{A}}=a_s e^{i\theta_s} a_w e^{-i\theta_w}, \quad \overline{\mathcal{B}}=b_s e^{i\delta_s} b_w e^{-i\delta_w} \quad . \tag{1.17}$$

The rate difference is

$$\begin{aligned}\Gamma-\overline{\Gamma} &= |\mathcal{A}+\mathcal{B}|^2-|\overline{\mathcal{A}}+\overline{\mathcal{B}}|^2 && (1.18)\\ &= 2a_s a_w b_s b_w \sin(\delta_s-\theta_s)\sin(\delta_w-\theta_w) \quad . && (1.19)\end{aligned}$$

A weak phase difference is guaranteed in the appropriate decay mode (different CKM phases), but the strong phase difference is not; it is very difficult to predict the magnitude of strong phase differences.

As an example consider the possibility of observing CP violation by measuring a rate difference between $B^- \to K^-\pi^o$ and $B^+ \to K^+\pi^o$. The $K^-\pi^o$ final state can be reached either by tree or penguin diagrams as shown in Fig. 1.5. The tree diagram has an imaginary

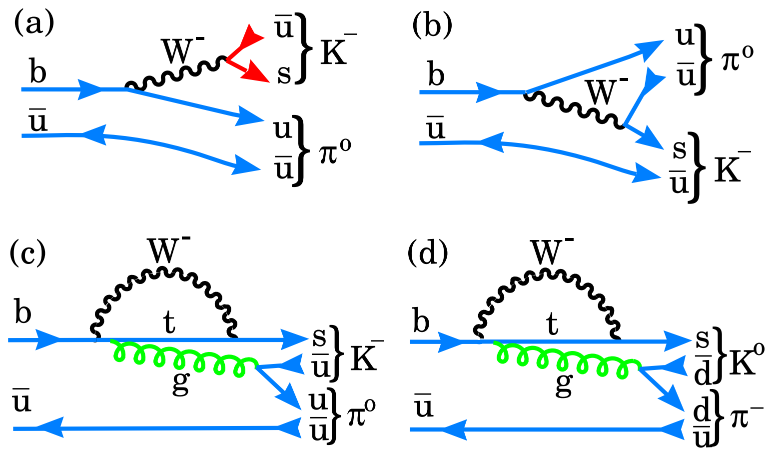

Figure 1.5: Diagrams for $B^- \to K^-\pi^o$, (a) and (b) are tree level diagrams where (b) is color suppressed; (c) is a penguin diagram. (d) Shows a penguin diagram for $B^- \to K^o\pi^-$, which cannot be produced via a tree diagram.

part coming from the V_{ub} coupling, while the penguin term does not, thus insuring a weak phase difference. This type of CP violation is called "direct." Note also that the process $B^- \to K^o\pi^-$ can only be produced by the penguin diagram in Fig. 1.5(d). Therefore, in this simple example, we do not expect a rate difference between $B^- \to K^o\pi^-$ and $B^+ \to K^o\pi^+$. (There have been suggestions that rescattering effects may contribute here and produce a rate asymmetry, see section 1.8.)

1.4 CP Violation Formalism in Neutral B decays

For neutral mesons we can construct the CP eigenstates

$$|B_1^o\rangle = \frac{1}{\sqrt{2}}\left(|B^o\rangle + |\overline{B}^o\rangle\right) \quad , \tag{1.20}$$

$$|B_2^o\rangle = \frac{1}{\sqrt{2}}\left(|B^o\rangle - |\overline{B}^o\rangle\right) \quad , \tag{1.21}$$

where

$$CP|B_1^o\rangle = |B_1^o\rangle \quad , \tag{1.22}$$

$$CP|B_2^o\rangle = -|B_2^o\rangle \quad . \tag{1.23}$$

Since B^o and $\overline{B}^o$ can mix, the mass eigenstates are superpositions of $a|B^o\rangle + b|\overline{B}^o\rangle$ which obey the Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} a \\ b \end{pmatrix}. \tag{1.24}$$

If CP is not conserved then the eigenvectors, the mass eigenstates $|B_L\rangle$ and $|B_H\rangle$, are not the CP eigenstates but are

$$|B_L\rangle = p|B^o\rangle + q|\overline{B}^o\rangle, \quad |B_H\rangle = p|B^o\rangle - q|\overline{B}^o\rangle, \tag{1.25}$$

where

$$p = \frac{1}{\sqrt{2}}\frac{1+\epsilon_B}{\sqrt{1+|\epsilon_B|^2}}, \quad q = \frac{1}{\sqrt{2}}\frac{1-\epsilon_B}{\sqrt{1+|\epsilon_B|^2}}. \tag{1.26}$$

CP is violated if $\epsilon_B \neq 0$, which occurs if $|q/p| \neq 1$.

The time dependence of the mass eigenstates is

$$|B_L(t)\rangle = e^{-\Gamma_L t/2}e^{-im_L t/2}|B_L(0)\rangle \tag{1.27}$$
$$|B_H(t)\rangle = e^{-\Gamma_H t/2}e^{-im_H t/2}|B_H(0)\rangle, \tag{1.28}$$

leading to the time evolution of the flavor eigenstates as

$$|B^o(t)\rangle = e^{-\left(im+\frac{\Gamma}{2}\right)t}\left(\cos\frac{\Delta mt}{2}|B^o(0)\rangle + i\frac{q}{p}\sin\frac{\Delta mt}{2}|\overline{B}^o(0)\rangle\right) \tag{1.29}$$

$$|\overline{B}^o(t)\rangle = e^{-\left(im+\frac{\Gamma}{2}\right)t}\left(i\frac{p}{q}\sin\frac{\Delta mt}{2}|B^o(0)\rangle + \cos\frac{\Delta mt}{2}|\overline{B}^o(0)\rangle\right), \tag{1.30}$$

where $m = (m_L + m_H)/2$, $\Delta m = m_H - m_L$ and $\Gamma = \Gamma_L \approx \Gamma_H$. Note that the fraction of B^o remaining at time t is given by $\langle B^o(t)|B^o(t)\rangle^*$, and is a pure exponential, $e^{-\Gamma t}$, in the absence of CP violation.

Indirect CP violation in the neutral B system

As in the case of K_L decay, we can look for the rate asymmetry

$$a_{sl} = \frac{\Gamma\left(\overline{B}^o(t) \to X\ell^+\nu\right) - \Gamma\left(B^o(t) \to X\ell^-\overline{\nu}\right)}{\Gamma\left(\overline{B}^o(t) \to X\ell^+\nu\right) + \Gamma\left(B^o(t) \to X\ell^-\overline{\nu}\right)} \tag{1.31}$$

$$= \frac{1-\left|\frac{q}{p}\right|^4}{1+\left|\frac{q}{p}\right|^4} \approx O\left(10^{-3}\right). \tag{1.32}$$

These final states occur only through mixing as the direct decay occurs only as $B^o \to X\ell^+\nu$. To generate CP violation we need an interference between two diagrams. In this case the two diagrams are the mixing diagram with the t-quark and the mixing diagram with the c-quark. This is identical to what happens in the K_L^o case. This type of CP violation is called "indirect." The small size of the expected asymmetry is caused by the off-diagonal elements of the Γ matrix in equation (1.24) being very small compared to the off-diagonal elements of the mass matrix, i.e. $|\Gamma_{12}/M_{12}| \ll 1$ and $\mathrm{Im}(\Gamma_{12}/M_{12}) \neq 0$. This results from the nearly equal widths of the B_L^o and B_H^o [22].

In the case of the B_s^o a relatively large, ≈15% component of B_s decays is predicted to end up as a $c\bar{c}s\bar{s}$ final state. Since $\overline{B}_s$ decays with the same rate into the same final state, it has been predicted [23, 24, 25] that there will be a substantial width difference $\Delta\Gamma = \Gamma_H - \Gamma_L \approx 15\%\Gamma$, between CP+ and CP- eigenstates. BTeV can easily measure this lifetime difference by measuring the lifetime of a mixed CP state such as $D_s^+\pi^-$ and comparing with the CP+ state $J/\psi\eta'$. The CP+ state K^+K^- can also be used [26]. For finite $\Delta\Gamma$, equations 1.29 and 1.30 are modified [27]. See section 1.8.5 for more details.

CP violation for B via interference of mixing and decays

Here we choose a final state f which is accessible to both B^o and $\overline{B}^o$ decays. The second amplitude necessary for interference is provided by mixing. Fig. 1.6 shows the decay into f either directly or indirectly via mixing. It is necessary only that f be accessible from either

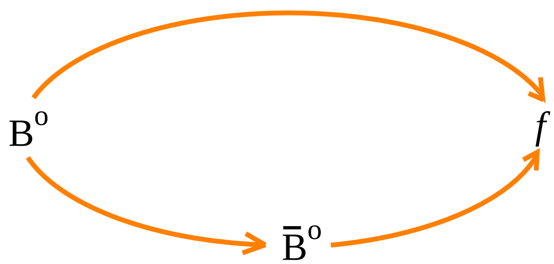

Figure 1.6: Two interfering ways for a B^o to decay into a final state f.

state. However if f is a CP eigenstate the situation is far simpler. For CP eigenstates

$$CP|f_{CP}\rangle = \pm|f_{CP}\rangle. \tag{1.33}$$

It is useful to define the amplitudes

$$A = \langle f_{CP}|\mathcal{H}|B^o\rangle, \quad \bar{A} = \langle f_{CP}|\mathcal{H}|\overline{B}^o\rangle. \tag{1.34}$$

If $\left|\frac{\bar{A}}{A}\right| \neq 1$, then we have "direct" CP violation in the decay amplitude, which was discussed above. Here CP can be violated by having

$$\lambda = \frac{q}{p}\cdot\frac{\bar{A}}{A} \neq 1, \tag{1.35}$$

which requires only that λ^2 acquire a non-zero phase, i.e. $|\lambda|$ could be unity and CP violation can occur.

The asymmetry, in this case, is defined as

$$a_{f_{CP}} = \frac{\Gamma\left(B^o(t) \to f_{CP}\right) - \Gamma\left(\overline{B}^o(t) \to f_{CP}\right)}{\Gamma\left(B^o(t) \to f_{CP}\right) + \Gamma\left(\overline{B}^o(t) \to f_{CP}\right)}, \tag{1.36}$$

which for $|q/p| = 1$ gives

$$a_{f_{CP}} = \frac{(1-|\lambda|^2)\cos\left(\Delta m t\right) - 2\mathrm{Im}\lambda\sin(\Delta m t)}{1+|\lambda|^2}. \tag{1.37}$$

For the cases where there is only one decay amplitude A, $|\lambda|$ equals 1, and we have

$$a_{f_{CP}} = -\mathrm{Im}\lambda\sin(\Delta m t). \tag{1.38}$$

Only the amplitude, $-\mathrm{Im}\lambda$ contains information about the level of CP violation, the sine term is determined only by B^o mixing. In fact, the time integrated asymmetry is given by

$$a_{f_{CP}} = -\frac{x}{1+x^2}\mathrm{Im}\lambda \quad , \tag{1.39}$$

where $x = \frac{\Delta m}{\Gamma}$. For the case of the B_d^o, $x/(1+x^2)$= 0.48, which is quite lucky as the maximum size of the coefficient is -0.5.

$\mathrm{Im}\lambda$ is related to the CKM parameters. Recall $\lambda = \frac{q}{p}\cdot\frac{\bar{A}}{A}$. The first term is the part that comes from mixing:

$$\frac{q}{p} = \frac{\left(V_{tb}^* V_{td}\right)^2}{\left|V_{tb}V_{td}\right|^2} = \frac{\left(1-\rho-i\eta\right)^2}{\left(1-\rho+i\eta\right)\left(1-\rho-i\eta\right)} = e^{-2i\beta} \quad \text{and} \tag{1.40}$$

$$\mathrm{Im}\frac{q}{p} = -\frac{2(1-\rho)\eta}{\left(1-\rho\right)^2+\eta^2} = \sin(2\beta). \tag{1.41}$$

To evaluate the decay part we need to consider specific final states. For example, consider $f \equiv \pi^+\pi^-$. The simple spectator decay diagram is shown in Fig. 1.7. For the moment we will assume that this is the only diagram which contributes. Later we will show why this is not true. For this $b \to u\bar{u}d$ process we have

$$\frac{\bar{A}}{A} = \frac{\left(V_{ud}^* V_{ub}\right)^2}{\left|V_{ud}V_{ub}\right|^2} = \frac{(\rho-i\eta)^2}{(\rho-i\eta)(\rho+i\eta)} = e^{-2i\gamma}, \tag{1.42}$$

and

$$\mathrm{Im}(\lambda) = \mathrm{Im}(e^{-2i\beta}e^{-2i\gamma}) = \mathrm{Im}(e^{2i\alpha}) = \sin(2\alpha) \quad . \tag{1.43}$$

[2] λ here is not the same variable that occurs in the Wolfenstein representation of the CKM matrix.

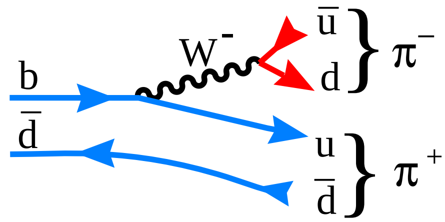

Figure 1.7: Decay diagram at the tree level for $B^o \to \pi^+\pi^-$.

The final state $J/\psi K_S$ plays an especially important role in the study of CP violation. It is a CP eigenstate and its decay is dominated by only one diagram, shown in Fig. 1.8. In this case we do not get a phase from the decay part because

$$\frac{\bar{A}}{A} = \frac{(V_{cb}V_{cs}^*)^2}{|V_{cb}V_{cs}|^2} \tag{1.44}$$

is real. In this case the final state is a state of negative CP, i.e. $CP|J/\psi K_S\rangle = -|J/\psi K_S\rangle$.

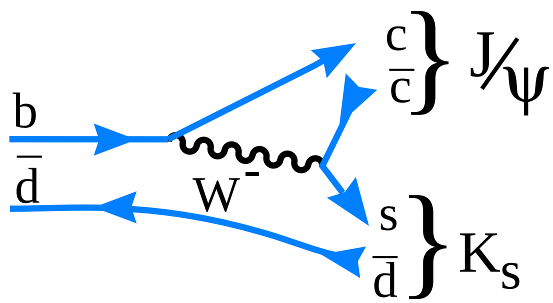

Figure 1.8: Decay diagram at the tree level for $B^o \to J/\psi K_S$.

This introduces an additional minus sign in the result for Imλ. Before finishing discussion of this final state we need to consider in more detail the presence of the K_S in the final state. Since neutral kaons can mix, we pick up another mixing phase. This term creates a phase given by

$$\left(\frac{q}{p}\right)_K = \frac{(V_{cd}^*V_{cs})^2}{|V_{cd}V_{cs}|^2}, \tag{1.45}$$

which is zero. It is necessary to include this term, however, since there are other formulations of the CKM matrix than Wolfenstein, which have the phase in a different location. It is important that the physics predictions not depend on the CKM convention.[3]

In summary, for the case of $f = J/\psi K_S$, Im$\lambda = -\sin(2\beta)$.

[3]Here we don't include CP violation in the neutral kaon since it is much smaller than what is expected in the B decay.

1.5 Techniques for Determining β

The decay $B^o \rightarrow J/\psi K_S$ is the primary source for measurements of $\sin(2\beta)$. In the common phase convention, CP violation is expected to arise mostly from the mixing, driven by $\mathrm{Im}(q/p)$, while the decay amplitude, $\mathrm{Im}(\bar{A}/A)$, is expected to contribute only a small part (see Fig. 1.8).[4]

1.5.1 Results on $\sin 2\beta$

For years observation of large CP violation in the B system was considered to be one of the corner stone predictions of the Standard Model. Yet it took a very long time to come up with definitive evidence. The first statistically significant measurements of CP violation in the B system were made recently by BABAR and BELLE [2]. This enormous achievement was accomplished using an asymmetric e^+e^- collider on the $\Upsilon(4S)$ which was first suggested by Pier Oddone. The measurements are listed in Table 1.1, along with other previous indications [12].

Table 1.1: Measurements of $\sin 2\beta$.

Experiment	$\sin 2\beta$
BABAR	$0.59 \pm 0.14 \pm 0.05$
BELLE	$0.99 \pm 0.14 \pm 0.06$
Average	$0.79{\pm}0.11$
CDF	$0.79^{+.41}_{-0.44}$
ALEPH	$0.84^{+0.82}_{-1.04} \pm 0.16$
OPAL	$3.2^{+1.8}_{-2.0} \pm 0.5$

The average value of 0.79 ± 0.11 is taken from BABAR and BELLE only. These two measurements do differ by a sizeable amount, but the confidence level that they correctly represent the same value is 6%. This value is consistent with what is expected from the other known constraints on ρ and η. We have

$$\overline{\eta} = (1-\overline{\rho})\frac{1 \pm \sqrt{1-\sin^2 2\beta}}{\sin 2\beta} \quad . \tag{1.46}$$

There is a four fold ambiguity in the translation between $\sin 2\beta$ and the linear constraints in the $\rho - \eta$ plane. These occur at β, $\pi/2 - \beta$, $\pi + \beta$ and $3\pi/2 - \beta$. Two of these constraints are shown in Figure 1.9. The other two can be viewed by extending these to negative $\overline{\eta}$. We think $\eta > 0$ based only on measurement of ϵ' in the neutral kaon system. This analysis clearly shows that current data are consistent with the Standard Model.

[4]Actually the only phase that has physical meaning is the product of $q/p \cdot \bar{A}/A$.

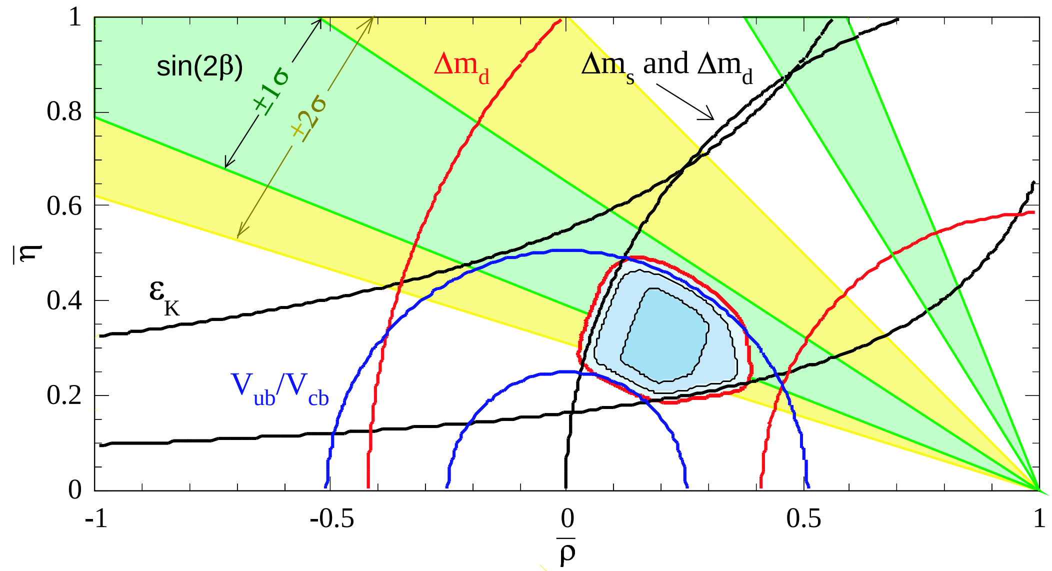

Figure 1.9: Constraints from $\sin 2\beta$ measurement overlaid with other constraints from Hocker *et al.* [16]. The inner band is at 1σ while the outer band, shown on one band only, is at 2σ.

In BTeV, we aim to improve significantly on the precision of the $\sin(2\beta)$ measurement. Furthermore, we intend to be able to remove "ambiguities." When we measure $\sin(2\phi)$, where ϕ is any angle, we have a four-fold ambiguity in ϕ, namely ϕ, $\pi/2-\phi$, $\phi+\pi$ and $3\pi/2-\phi$. These ambiguities can mask the effects of new physics. Our task is to remove as many of the ambiguities as possible.

1.5.2 Removal of Two of the β Ambiguities

The decay $B\to J/\psi\, K^*(890)$, where $K^*\to K_S\pi^o$ can be used to get information about the sign of $\cos(2\beta)$, which would remove two of the ambiguities [28]. This decay is described by three complex decay amplitudes. Following a suggestion of Dighe, Dunietz, and Fleischer [29, 30], we write the decay amplitudes $A_0=-\sqrt{1/3}\,S+\sqrt{2/3}\,D$, $A_\parallel=\sqrt{2/3}\,S+\sqrt{1/3}\,D$, and $A_\perp=P$, where S, P, and D denote S, P, and D wave amplitudes, respectively. Normalizing the decay amplitudes to $|A_0|^2+|A_\parallel|^2+|A_\perp|^2=1$ and eliminating one overall phase leaves four independent parameters.

The full angular distribution of a B meson decaying into two vector particles is specified by three angles. The helicity angle basis [31] has been used for angular analyses of $B\to J/\psi\, K^*$ decays. An alternative basis, called the transversity basis is more suitable for extracting parity information [30].

In the transversity basis, the direction of the K^* in the J/ψ rest frame defines the x-axis of a right-handed coordinate system. The $K\pi$ plane fixes the y-axis with $p_y(K)>0$ and the normal to this plane defines the z-axis. The transversity angles $\theta_{\rm tr}$ and $\phi_{\rm tr}$ are then defined as polar and azimuth angles of the l^+ in the J/ψ rest frame. The third angle, the

K^* decay angle θ_{K^*}, is defined as that of the K in the K^* rest frame relative to the negative of the J/ψ direction in that frame. Using these definitions the full angular distribution of the $B \to J/\psi\, K^*$ decay is [30]:

$$
\begin{aligned}
&\frac{1}{\Gamma}\,\frac{\mathrm{d}^3\Gamma}{\mathrm{d}\cos\theta_{\rm tr}\;\mathrm{d}\cos\theta_{K^*}\;\mathrm{d}\phi_{\rm tr}} \\
&\quad = \tfrac{9}{32\pi}\,\{2\,|A_0|^2\;\cos^2\theta_{K^*}(1-\sin^2\theta_{\rm tr}\cos^2\phi_{\rm tr}) \\
&\qquad\quad + |A_\parallel|^2\;\sin^2\theta_{K^*}(1-\sin^2\theta_{\rm tr}\sin^2\phi_{\rm tr}) \\
&\qquad\quad + |A_\perp|^2\;\sin^2\theta_{K^*}\sin^2\theta_{\rm tr}\sin^2\phi_{\rm tr} \\
&\qquad\quad - \mathrm{Im}\,(A_\parallel^* A_\perp)\;\sin^2\theta_{K^*}\sin 2\theta_{\rm tr}\sin\phi_{\rm tr} \\
&\qquad\quad + \tfrac{1}{\sqrt{2}}\,\mathrm{Re}\,(A_0^* A_\parallel)\;\sin 2\theta_{K^*}\sin^2\theta_{\rm tr}\sin 2\phi_{\rm tr} \\
&\qquad\quad + \tfrac{1}{\sqrt{2}}\,\mathrm{Im}\,(A_0^* A_\perp)\;\sin 2\theta_{K^*}\sin 2\theta_{\rm tr}\cos\phi_{\rm tr}\;\}.
\end{aligned}
\tag{1.47}
$$

For $\bar{B}$ decays the interference terms containing $A_\perp$ switch sign while all other terms remain unchanged.

Results shown in Table 1.2 have been obtained from CLEO, CDF and BABAR using the decay $\bar{K}^{*o} \to K^-\pi^+$.

Parameter	CLEO [32]	CDF [33]	BABAR [34]
$\lvert A_0\rvert^2 \;=\; \Gamma_{\rm L}\,/\,\Gamma$	$0.52 \pm 0.07 \pm 0.04$	$0.59 \pm 0.06 \pm 0.02$	$0.60 \pm 0.03 \pm 0.02$
$\lvert A_\perp\rvert^2 \;=\; \lvert P\rvert^2$	$0.16 \pm 0.08 \pm 0.04$	$0.13\,^{+0.12}_{-0.06} \pm 0.03$	$0.16 \pm 0.03 \pm 0.01$

Table 1.2: Resulting decay amplitudes from the fit to the transversity angles. The first error is statistical and the second is the estimated systematic uncertainty.

The parity odd component, $|A_\perp|^2$, has been definitely established by BABAR as being significantly non-zero, and is ≈25% of the rate of the parity even component. This is likely large enough to allow the determination of the sign of the interference terms using the tagged $K^{*o} \to K_S\pi^o$ decays; that, in turn, allows a determination of the sign of the product of $\cos(2\beta)$ with a strong phase-shift. The sign of this phase-shift can either be obtained from factorization, which is a dangerous procedure, or using the much weaker assumption of SU(3) symmetry, and analyzing the time-dependent oscillations in the decay $B_s \to J/\psi\phi$ [28], where the mixing phase is expected to be small.

Another independent method of removing two of the ambiguities is to measure the sign of the $\cos(2\beta)$ term in the decay $B^o \to J/\psi K^o$, $K^o \to \pi^\pm\ell^\mp\nu$. This idea developed by Kayser [35], works because of the interference between K_L and K_S in the decay, where the decay amplitudes are equal. The time evolution of the decay width can be expressed in terms of the B^o decay time (t_B) and the K^o decay time (t_K) as

$$
\begin{aligned}
\Gamma(t_B,t_K)\propto & \\
& e^{-\Gamma_B t_B}\Big\{e^{-\gamma_s t_K}\left[1\mp sin(2\beta)sin(\Delta m_B t_B)\right] \\
& \quad +e^{-\gamma_L t_K}\left[1\pm sin(2\beta)sin(\Delta m_B t_B)\ \right] \\
& \quad \pm(\mp)2e^{-\frac{1}{2}(\gamma_s+\gamma_L)t_K}\big[\ \cos(\Delta m_B t_B)\cos(\Delta m_K t_K) \\
& \qquad\qquad +\cos(2\beta)sin(\Delta m_B t_B)sin(\Delta m_K t_K)\big]\ \Big\},
\end{aligned}
\tag{1.48}
$$

where the top sign of each pair is for B^o, and the bottom for $\overline{B}^o$. The first pair of signs in the third line refers to the kaon decay mode $\pi^-\ell^+\nu$ (K), while the second pair is for $\pi^+\ell^-\bar{\nu}$ ($\overline{K}$).

To get an idea of the predicted asymmetries, we integrate this equation over t_B. There are four different rates that can be denoted as combinations of B and $\overline{B}$ with K and $\overline{K}$. In Fig. 1.10 we show the four rates as solid lines if $\cos(2\beta)$ were positive and the four rates as dashed lines if $\cos(2\beta)$ were negative. These were done for $\sin(2\beta) = 0.7$. If $\sin(2\beta)$ were smaller the rate differences would be larger and vice-versa.

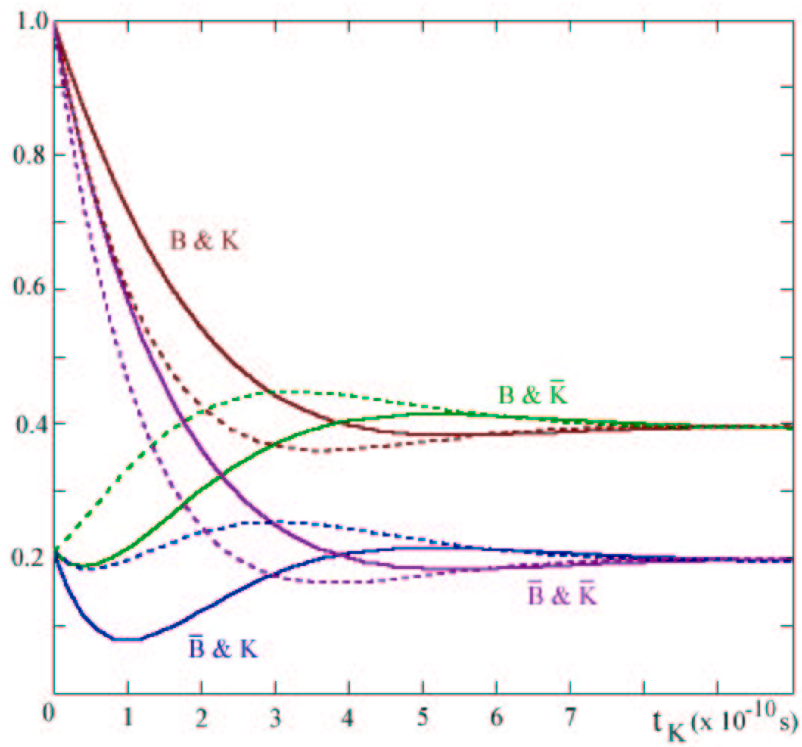

Figure 1.10: The decay rates for $B^o \to J/\psi K^o$, $K^o \to \pi\ell\nu$, as a function of K^o decay time, integrated over the B^o decay time. The solid lines have the sign of the $\cos(2\beta)$ term as positive, while the corresponding dashed lines have negative values. The absolute normalization is arbitrary, and $\sin(2\beta)$ was fixed at 0.7.

The differences are large over about five K_S lifetimes. Since only the sign of the $\cos(2\beta)$

term needs to be found, all other parameters, including $\sin(2\beta)$ are specified. Unfortunately, the event rate is rather small, since $\mathcal{B}(K_S \to \pi\ell\nu) = 1.4 \times 10^{-3}$ and although $\mathcal{B}(K_L \to \pi\ell\nu) = 0.66$, only 1% of the K_L decay soon enough to be of use. Roughly, we have about 100 times fewer events than in $J/\psi K_S$. However, if the backgrounds are not too large, it will only take on the order of a hundred events to successfully determine the sign of $\cos(2\beta)$ using this technique.

It is interesting to note that measuring this combination of B^o and K^o decay modes can lead to measurements of CPT violation [36].

1.5.3 Other Modes for Measuring sin(2β)

New physics can add differently to the phases in different decay modes if it contributes differently to the relative decay amplitudes $A/\overline{A}$. Therefore it is interesting to measure CP violation in redundant modes. For example, the decay $B^o \to \phi K_S$ should also measure $\sin(2\beta)$. If it is different than that obtained by $B^o \to J/\psi K_S$, that would be a strong indication of new physics [37]. We list in Table 1.3 other interesting modes to check $\sin(2\beta)$. The branching ratios listed with errors have been measured [38, 39, 40], while those without are theoretical estimates.

Table 1.3: Other modes useful for cross-checking $\sin(2\beta)$

Decay Mode	Branching Ratio
$B^o \to \phi K_S$	$(4.0 \pm 2.1)) \times 10^{-6}$
$B^o \to D^+D^-$	$\approx 10^{-3}$
$B^o \to D^{*+}D^-$	$\approx 10^{-3}$
$B^o \to \eta\prime K^o$,	$(5.9 \pm 1.9) \times 10^{-5}$
$B^o \to J/\psi(\pi^o,\ \eta$ or $\eta\prime)$	$(3.4 \pm 1.6) \times 10^{-5}$

1.6 Comment on Penguin Amplitudes

Many processes can have penguin components. The diagram for $B^o \to \pi^+\pi^-$ is shown in Fig. 1.11. The $\pi^+\pi^-$ final state is expected to have a rather large penguin amplitude ~20% of the tree amplitude. Then $|\lambda| \neq 1$ and $a_{f_{CP}}$, equation 1.37, develops a $\cos(\Delta mt)$ term. In the $J/\psi K_S$ case, the penguin amplitude is expected to be small since a $c\bar{c}$ pair must be "popped" from the vacuum; even if the penguin decay amplitude were of significant size here, the decay phase, $\mathrm{Im}(\overline{A}/A)$ is the same as the tree level process, and quite small.

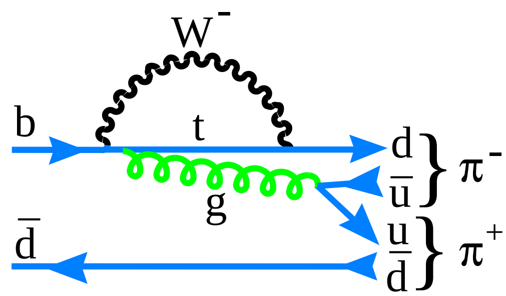

Figure 1.11: Penguin diagram for $B^o \to \pi^+\pi^-$.

1.7 Techniques for Determining α

1.7.1 Introduction

Measuring α is more difficult than measuring β in several respects. First of all, the decay amplitudes are modulated by V_{ub} rather than V_{cb}, making the overall rates small. Secondly, the gluonic penguin rates are of the same order causing well known difficulties in extracting the weak phase angle (see section 1.6 above). The penguin diagrams add a third amplitude to the tree level and mixing amplitudes. It turns out, however, that this complication can be a blessing in disguise. The interference generates $\cos(2\alpha)$ terms in the decay rate, that can be used to remove discrete ambiguities.

The decay $B^o \to \pi^+\pi^-$ has oft been cited as a way to measure $\sin(2\alpha)$. However, the penguin pollution mentioned above, makes it difficult to extract the angle. Table 1.4 lists the currently measured branching ratios for B decays into $\pi\pi$ or $K\pi$.

Table 1.4: Branching Ratios for $B \to K\pi$ and $B \to \pi\pi$ in units of 10^{-6}.

Mode	CLEO [41]	BABAR [42]	BELLE [43]	Average
$\pi^+\pi^-$	$4.7^{+1.8}_{-1.5} \pm 0.6$	$4.1 \pm 1.0 \pm 0.7$	$5.6^{+2.3}_{-2.0} \pm 0.4$	$4.5^{+0.9}_{-0.8}$
$\pi^+\pi^o$	<12	<9.6	<13.4	
$K^\pm\pi^\mp$	$18.8^{+2.8}_{-2.6} \pm 1.3$	$16.7 \pm 1.6 \pm 1.3$	$19.3^{+3.4+1.5}_{-3.2-.06}$	$17.7^{+1.6}_{-1.5}$
$K^+\pi^o$	$12.1^{+3.0+2.1}_{-2.8-1.4}$	$10.8^{+2.1}_{-1.9} \pm 1.6$	$16.3^{+3.5+1.6}_{-3.3-1.8}$	$12.1^{+1.7}_{-1.6}$
$K^o\pi^-$	$18.2^{+4.6}_{-4.0} \pm 1.6$	$18.2^{+3.3}_{-3.0} \pm 2.0$	$13.7^{+5.7+1.9}_{-4.8-1.8}$	$17.3^{+2.7}_{-2.4}$
$K^o\pi^o$	$14.8^{+5.9+2.4}_{-5.1-3.3}$	$8.2^{+3.1}_{-2.7} \pm 1.2$	$16.0^{+7.2+2.5}_{-5.9-2.7}$	$10.4^{+2.7}_{-2.5}$

These results indicate that the penguin amplitude is quite large and cannot be ignored. Gronau and London [44] have shown that an isospin analysis using the additional decays $B^- \to \pi^-\pi^o$ and $B^o \to \pi^o\pi^o$ can be used to extract α [45], but the $\pi^o\pi^o$ final state is extremely difficult to detect in any existing or proposed experiment. Other authors have suggested different methods [46], but they all have theoretical assumptions. Thus, measurement of the

CP asymmetry in $B^o \rightarrow \pi^+\pi^-$ cannot, in our view, provide an accurate determination of $\sin(2\alpha)$ unless some new breakthrough in theory occurs.

1.7.2 Using $B^o \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^o$ To Determine α

There is however, a theoretically clean method to determine α. The interference between tree and penguin diagrams can be exploited by measuring the time dependent CP violating effects in the decays $B^o \rightarrow \rho\pi$ as shown by Snyder and Quinn [47]. There are three such neutral decay modes, listed in Table 1.5 with their respective penguin and tree amplitudes, denoted by T^{ij}, where i lists charge of the ρ and j the charge of the π. For the $\rho^o\pi^o$ mode, isospin constraints are used to eliminate T^{oo}. The amplitudes for the charged decays are also given.

Table 1.5: $B^o \rightarrow \rho\pi$ Decay Modes

Decay Mode	Decay Amplitudes
$\sqrt{2}A(B^+ \rightarrow \rho^+\pi^o)$	$=S_1 = T^{+o} + 2P_1$
$\sqrt{2}A(B^+ \rightarrow \rho^o\pi^+)$	$=S_2 = T^{o+} - 2P_1$
$A(B^o \rightarrow \rho^+\pi^-)$	$=S_3 = T^{+-} + P_1 + P_o$
$A(B^o \rightarrow \rho^-\pi^+)$	$=S_4 = T^{-+} - P_1 + P_o$
$2A(B^o \rightarrow \rho^o\pi^o)$	$=S_5 = T^{+-} + T^{+-} - T^{+o} - T^{o+} - 2P_o$

For the $\rho\pi$ final state, the ρ decay amplitude can be parameterized as

$$f(m,\theta) = \frac{\cos(\theta)\Gamma_\rho}{2(m_\rho - m - i0.5\Gamma_\rho)} \quad , \tag{1.49}$$

where m_ρ is the ρ mass of 0.77 GeV and Γ_ρ, the width of 0.15 GeV. θ is the helicity decay angle and the $\cos(\theta)$ dependence arises because the ρ must be fully polarized in this decay which starts with a spin-0 B and ends with a spin-1 ρ and spin-0 π.

The full decay amplitudes for $B^o \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^o$ and the corresponding $\overline{B}^o$ decay are given by

$$\begin{aligned} A(B^o) &= f^+S_3 + f^-S_4 + f^oS_5/2 \\ A(\overline{B}^o) &= f^+\overline{S}_3 + f^-\overline{S}_4 + f^o\overline{S}_5/2 \quad , \end{aligned} \tag{1.50}$$

where the superscript on the f indicates the charge of the ρ. The sum over the three neutral B decay amplitudes involves only tree amplitudes; the penguins vanish. The angle between this sum for B^o decays ($\equiv T$) and the sum for $\overline{B}^o$ ($\equiv \overline{T}$) is precisely α. Computing the amplitudes gives a series of terms which have both $\sin(\Delta mt)$ and $\cos(\Delta mt)$ time dependences and coefficients which depend on both $\sin(2\alpha)$ and $\cos(2\alpha)$.

To extract α only the neutral modes need be measured. Further constraints and information about penguin phases can be extracted if the charged B's are also measured. But this is difficult because there are two π^o's in the $\rho^+\pi^o$ decay mode.

The $\rho\pi$ final state has many advantages. First of all, it has a relatively large branching ratio. The CLEO measurement for the $\rho^o\pi^+$ final state is $(1.0\pm0.3\pm0.2)\times10^{-5}$ [48]. The rate for the neutral B final state $\rho^\pm\pi^\mp$ is $(2.8^{+0.8}_{-0.7}\pm0.4)\times10^{-5}$, while the $\rho^o\pi^o$ final state is limited at 90% confidence level to $< 5.1 \times 10^{-6}$ [49]. BABAR finds a quite similar rate for $\rho^\pm\pi^\mp$ of $(2.9\pm0.5\pm0.4)\times10^{-5}$, and limits $\rho^o\pi^o$ at 90% confidence level to $< 10.6\times10^{-6}$ [42]. These measurements are consistent with theoretical expectations [50]. Secondly, since the ρ is fully polarized, the periphery of the Dalitz plot to be heavily populated, especially the corners. A sample Dalitz plot is shown in Fig. 1.12. This kind of distribution is good for maximizing the interferences, which helps minimize the error. Furthermore, little information is lost by excluding the Dalitz plot interior, a good way to reduce backgrounds.

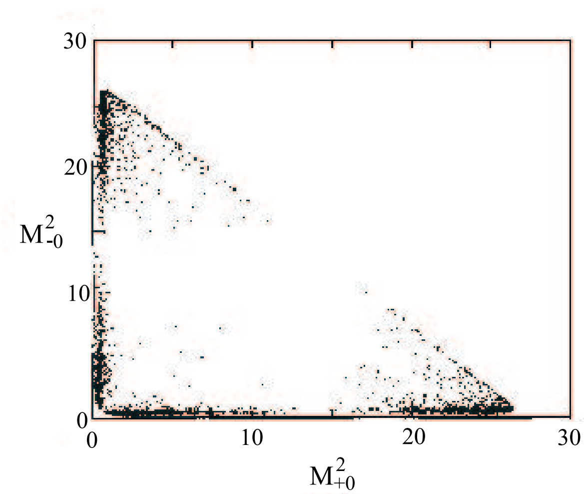

Figure 1.12: The Dalitz plot for $B^o \to \rho\pi \to \pi^+\pi^-\pi^o$ from Snyder and Quinn.

Snyder and Quinn have performed an idealized analysis that uses 1000 or 2000 flavor tagged background free events. The 1000 event sample usually yields good results for α, but sometimes does not resolve the ambiguity. With the 2000 event sample, however, they always succeed.

Recently Quinn and Silva have pointed out ways of using time integrated untagged data to specify some of the parameters with larger data samples [51]. Some concern for the effect of the B^* pole on the data has been expressed by Deandrea *et al.* [52].

1.7.3 Use of $B^o \to \pi^+\pi^-$ for Ambiguity Resolution

The decay $B^o \to \pi^+\pi^-$ can be used with some theoretical input to resolve the remaining ambiguity in $\sin(2\alpha)$. The difference in CP asymmetries between $\pi\pi$ and $\rho\pi$ is given by

$$a(\pi\pi) - a(\rho\pi) = -2(A_P/A_T)cos(\delta_P - \delta_T)\,[cos(2\alpha)\sin(\alpha)] \quad , \tag{1.51}$$

where A_P and A_T denote the penguin and tree amplitudes, respectively, and the δ's represent their strong phase shifts. Factorization can be used to get the sign of A_P/A_T and the strong phase shifts are believed to be small enough that $\cos(\delta_P - \delta_T)$ is positive [53].

1.8 Techniques for Determining γ

The angle γ could in principle be measured using a CP eigenstate of B_s decay that was dominated by the $b \to u$ transition. One such decay that has been suggested is $B_s \to \rho^o K_S$. However, there are the same "penguin pollution" problems as in $B^o \to \pi^+\pi^-$, but they are more difficult to resolve in the vector-pseudoscalar final state. (Note, the pseudoscalar-pseudoscalar final state here is $\pi^o K_S$, which does not have a measurable decay vertex.)

Fortunately, there are other ways of measuring γ. CP eigenstates are not used, which introduces discrete ambiguities. However, combining several methods should remove these. We have studied three methods of measuring γ.

1.8.1 Measurement of γ Using Time-Dependent CP violation in B_s Decays

The first method uses the decays $B_s \to D_s^{\pm} K^{\mp}$ where a time-dependent CP violation can result from the interference between the direct decays and the mixing-induced decays [54]. Fig. 1.13 shows the two direct decay processes for $\overline{B}_s^o$.

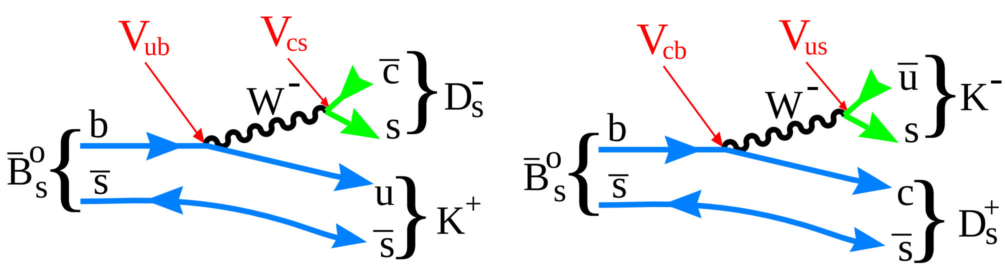

Figure 1.13: Two diagrams for $\overline{B}_s^o \to D_s^{\pm} K^{\mp}$.

Consider the following time-dependent rates that can be separately measured using flavor tagging of the other b:

$$\Gamma(B_s \to f) = |M|^2 e^{-t}\{\cos^2(xt/2) + \rho^2 \sin^2(xt/2) - \rho\sin(\phi + \delta)\sin(xt)\}$$
$$\Gamma(\bar{B}_s \to \bar{f}) = |M|^2 e^{-t}\{\cos^2(xt/2) + \rho^2 \sin^2(xt/2) + \rho\sin(\phi - \delta)\sin(xt)\}$$
$$\Gamma(B_s \to \bar{f}) = |M|^2 e^{-t}\{\rho^2\cos^2(xt/2) + \sin^2(xt/2) - \rho\sin(\phi - \delta)\sin(xt)\}$$
$$\Gamma(\bar{B}_s \to f) = |M|^2 e^{-t}\{\rho^2\cos^2(xt/2) + \sin^2(xt/2) + \rho\sin(\phi + \delta)\sin(xt)\}, \tag{1.52}$$

where $M = \langle\, f|B\rangle$, $\rho = \frac{\langle\, f|\bar{B}\rangle}{\langle\, f|B\rangle}$, ϕ is the weak phase between the 2 amplitudes and δ is the strong phase between the 2 amplitudes. The three parameters ρ, $\sin(\phi+\delta), \sin(\phi-\delta)$ can be extracted from a time-dependent study. If $\rho = O(1)$ the fewest number of events are required.

In the case of B_s decays where $f = D_s^+ K^-$ and $\bar{f} = D_s^- K^+$, the weak phase is γ.[5] Using this technique $\sin(\gamma)$ is determined with a four-fold ambiguity. If $\Delta\Gamma(B_s)$ is of the order of 10%, then the ambiguities can be directly resolved.

1.8.2 Measurement of γ Using Charged B Decay Rates

Another method for extracting γ has been proposed by Atwood, Dunietz and Soni [55], who refined a suggestion by Gronau and Wyler [56]. A large CP asymmetry can result from the interference of the decays $B^- \to K^- D^o, D^o \to f$ and $B^- \to K^- \overline{D}^o, \overline{D}^o \to f$, where f is a doubly-Cabibbo suppressed decay of the D^o (for example $f = K^+\pi^-, K\pi\pi$, etc.). The overall amplitudes for the two decays are expected to be approximately equal in magnitude. (Note that $B^- \to K^- \overline{D}^o$ is color-suppressed and $B^- \to K^- D^0$ is color-allowed.) The weak phase difference between them is γ. To observe a CP asymmetry there must also be a non-zero strong phase between the two amplitudes. It is necessary to measure the branching ratio $\mathcal{B}(B^- \to K^- f)$ for at least 2 different states f in order to determine γ up to discrete ambiguities. Three-body D^o decays are not suggested since the strong D decay phase shifts can vary over the Dalitz plot. Even in quasi-two body decays, such as $K^*\pi$ there may be residual interference effects which could lead to false results. Therefore, the modes that can best be used are $D^o \to K^-\pi^+$ and K^+K^- $(\pi^+\pi^-)$ final states.

We now discuss this method in more detail. Consider a two-body B^- decay into a neutral charmed meson, either a D^o or a $\overline{D}^o$ and a K^-. Let us further take the final state of the charmed meson to be a $K^+\pi^-$. There are two sequential decay processes that can lead to this situation, shown in Fig. 1.14. One is where the B^o decays into a D^o, that decays in a doubly-Cabibbo suppressed process. The other is where the B^o decays via a $b \to u$ transition to a D^o, that decays via a Cabibbo allowed process.

Remarkably, the decay rate for these two processes is quite similar leading to the possibility of large interference effects. Even if the interference effects are not large it is possible to use this method to determine γ, with some ambiguities. To see how this works, let us define the decay amplitudes and phases in Table 1.6 for two processes, one as described above and the other where the D^o or $\overline{D}^o$ decays into a CP eigenstate. (To be specific, we will take the K^+K^- final state.)

All quantities remain the same for the B^+ decays, except that the phase γ changes sign. The observed decay rates for the four processes can now be calculated by adding and squaring the amplitudes for the same final state. For example, the decay rate for $B^- \to [K^+\pi^-]\, K^-$ (where $[K^+\pi^-]$ denotes a $K^+\pi^-$ pair at the D^o mass), is given by

$$\Gamma(B^- \to \left[K^+\pi^-\right] K^-) = ac_d + bc + 2\sqrt{ac_d bc}\cos(\xi_1+\gamma) \quad , \tag{1.53}$$

[5]This is an approximation. The phase is precisely $\gamma - 2\chi + \chi'$, see section 1.12.2.

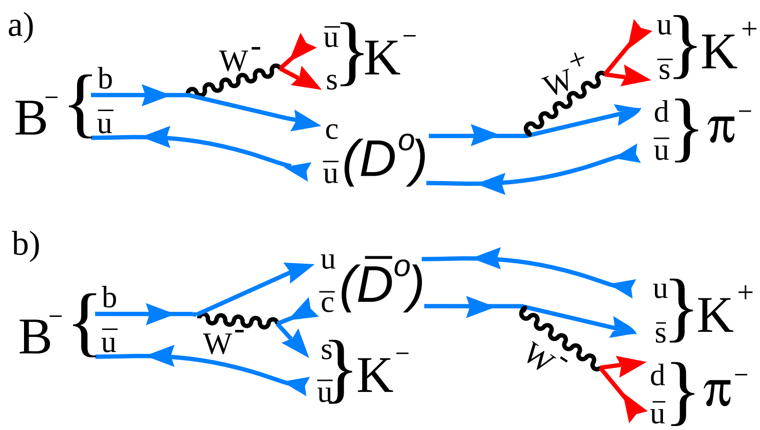

Figure 1.14: Diagrams for the two interfering processes, (a) $B^- \to D^o K^-$ (color allowed) followed by $D^o \to K^+\pi^-$ (doubly-Cabibbo suppressed), and (b) $B^- \to \overline{D}^o K^-$ (color suppressed) followed by $D^o \to K^-\pi^+$ (Cabibbo allowed).

Table 1.6: Amplitudes and Phases for $B^- \to D^o/\overline{D}^o K^-$

Decay Mode	B Amplitude	D Amplitude	Strong Phase	Weak Phase
$B^- \to D^o K^-$, $D^o \to K^+\pi^-$	$\sqrt{a}$	$\sqrt{c_d}$	$\delta_{B1} + \delta_{Cd}$	0
$B^- \to \overline{D}^o K^-$, $\overline{D}^o \to K^+\pi^-$	$\sqrt{b}$	$\sqrt{c}$	$\delta_{B2} + \delta_C$	γ
$B^- \to D^o K^-$, $D^o \to K^+K^-$	$\sqrt{a}$	$\sqrt{c_{CP}}$	$\delta_{B1} + \delta_{CP}$	0
$B^- \to \overline{D}^o K^-$, $\overline{D}^o \to K^-K^+$	$\sqrt{b}$	$\sqrt{c_{CP}}$	$\delta_{B2} + \delta_{CP}$	γ

where ξ_1 is a combination of B and D phase shifts, $\delta_{B2} - \delta_{B1} + \delta_C - \delta_{Cd}$ and is unknown. Similarly, the decay rates for the other processes are

$$\begin{aligned}
\Gamma(B^+ \to \left[K^-\pi^+\right] K^+) &= ac_d + bc + 2\sqrt{ac_d bc}\cos(\xi_1 - \gamma) \\
\Gamma(B^- \to \left[K^+K^-\right] K^-) &= ac_{CP} + bc_{CP} + 2\sqrt{abc_{CP}^2}\cos(\delta_B - \gamma) \\
\Gamma(B^+ \to \left[K^+K^-\right] K^+) &= ac_{CP} + bc_{CP} + 2\sqrt{abc_{CP}^2}\cos(\delta_B + \gamma)
\end{aligned} \tag{1.54}$$

where $\delta_B = \delta_{B1} - \delta_{B2}$.

In these four equations, the quantities which are known, or will be precisely known before this measurement is attempted are the decay widths a, c_d, c and c_{CP}. The unknowns are the decay width b, two strong phase shifts ξ_1 and δ_B and the weak phase shift γ. Thus the four equations may be solved for the four unknowns. We can find $\sin\gamma$ with a two-fold ambiguity. If more decay modes are added the ambiguity can be removed. The B^- decay mode can be changed from a K^- to a K^{*-}, which could change the strong B decay phase shift, or a different D^o decay mode can be used, such as $K^-\pi^+\pi^+\pi^-$, which would change the strong D decay phase shift. In the latter case, we have to worry about differences in strong phase shifts between D^o and $\overline{D}^o$ due to resonant structure, but use of this mode can shed some information on ambiguity removal.

Comparison of the solutions found here and using $B_s \to D_s^{\pm} K^{\mp}$ as described in the previous section are likely to remove the ambiguities.

1.8.3 Measurement of γ Using $B \to K\pi$ and $B \to \pi\pi$ Decay Rates and Asymmetries

The branching ratios into $\pi^+\pi^-$ and $K^{\pm}\pi^{\mp}$ shown in Table 1.4 can be used to get a very rough estimate of the ratio of penguin to tree contribution in the $\pi^+\pi^-$ final state. The $K\pi$ rate is about 4 times the $\pi\pi$ rate and is mostly penguin. Taking a Cabibbo suppression factor of ~ 16, we predict a penguin rate that is 25% of the tree rate in $\pi^+\pi^-$ and thus an amplitude that is about 50%. Therefore, the penguin and tree contributions for $B \to K\pi$ probably do not differ by more than a factor of four, so they can produce observable CP violating effects.

Several model dependent methods using the light two-body pseudoscalar decay rates have been suggested for measuring γ The basic idea in all these methods can be summarized as follows: $B^o \to \pi^+\pi^-$ has the weak decay phase γ. In order to reproduce the observed suppression of the decay rate for $\pi^+\pi^-$ relative to $K^{\pm}\pi^{\mp}$ we require a large negative interference between the tree and penguin amplitudes. This puts γ in the range of 90°. There is a great deal of theoretical work required to understand rescattering, form-factors etc...

Proposals for extracting information on γ have been made using the following experimental ratios:

$$
\begin{aligned}
R &= \frac{\tau(B^+)}{\tau(B^0)} \frac{\mathcal{B}(B^0 \to \pi^- K^+) + \mathcal{B}(\bar{B}^0 \to \pi^+ K^-)}{\mathcal{B}(B^+ \to \pi^+ K^0) + \mathcal{B}(B^- \to \pi^- \bar{K}^0)}, \\
R_* &= \frac{\mathcal{B}(B^+ \to \pi^+ K^0) + \mathcal{B}(B^- \to \pi^- \bar{K}^0)}{2[\mathcal{B}(B^+ \to \pi^0 K^+) + \mathcal{B}(B^- \to \pi^0 K^-)]},
\end{aligned}
\tag{1.55}
$$

The first, R, is by Fleischer and Mannel [57], and the second R_*, is by Neubert and Rosner [58], who updated an older suggestion of Gronau and Rosner [59]. The latter paper prompted much theoretical discussion about the effects of isospin conservation and rescattering [60, 61, 62, 63]. Neubert [64] takes into account these criticisms and provides a framework to limit γ.

More information is obtainable if the CP averaged $\pi^{\pm}\pi^o$ branching ratios are also measured, and a CP violating observable defined as

$$
\widetilde{A} \equiv \frac{A_{\rm CP}(\pi^0 K^+)}{R_*} - A_{\rm CP}(\pi^+ K^0) \quad , \tag{1.56}
$$

where for example

$$A_{\rm CP}(\pi^0 K^+) = \frac{\Gamma(B^+ \to \pi^o K^+) - \Gamma(B^- \to \pi^o K^-)}{\Gamma(B^+ \to \pi^o K^+) + \Gamma(B^- \to \pi^o K^-)} \quad . \tag{1.57}$$

To summarize Neubert's strategy for determining γ: From measurements of the CP-averaged branching ratio for the decays $B^\pm \to \pi^\pm \pi^0$, $B^\pm \to \pi^\pm K^0$ and $B^\pm \to \pi^0 K^\pm$, the ratio R_* and a parameter $\bar{\varepsilon}_{3/2}$ are determined. Next, from measurements of the rate asymmetries in the decays $B^\pm \to \pi^\pm K^0$ and $B^\pm \to \pi^0 K^\pm$ the quantity $\widetilde{A}$ is determined.

In Fig. 1.15, we show the contour bands as given by Neubert in the ϕ-γ plane. Here ϕ is a strong interaction phase-shift. Assuming that $\sin\gamma > 0$ as suggested by the global analysis of the unitarity triangle, the sign of $\widetilde{A}$ determines the sign of $\sin\phi$. In the plot, we assume here that $0^\circ \leq \phi \leq 180^\circ$. For instance, if $R_* = 0.7$ and $\widetilde{A} = 0.2$, then the two solutions are $(\gamma, \phi) \approx (98^\circ, 25^\circ)$ and $(\gamma, \phi) \approx (153^\circ, 67^\circ)$, only the first of which is allowed by the upper bound $\gamma < 105^\circ$ following from the global analysis of the unitarity triangle shown here (section 1.2 or in [16]). It is evident that the contours are rather insensitive to the rescattering effects. According to Neubert, the combined theoretical uncertainty is of order $\pm 10^\circ$ on the extracted value of γ.

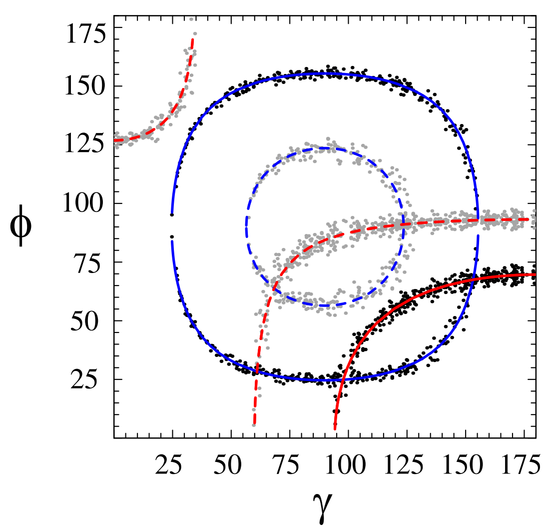

Figure 1.15: Contour plots from Neubert [64] for the quantities R_* ("hyperbolas") and $\widetilde{A}$ ("circles") plotted in the $\phi - \gamma$ plane. The units are degrees. The scatter plots show the results including rescattering effects, while the lines refer to $\varepsilon_a = 0$. The solid curves correspond to the contours for $R_* = 0.7$ and $\widetilde{A} = 0.2$, the dashed ones to $R_* = 0.9$ and $\widetilde{A} = 0.4$.

From the contour plots for the quantities R_* and $\widetilde{A}$ the phases γ and ϕ can then be extracted up to discrete ambiguities. There are also errors in theoretical parameters that must be accounted for.

Beneke *et al.* [65] have recently developed a sophisiticated model of QCD factorization with corrections. Fig. 1.16 shows how values of γ that can be found in their framework, once better data are obtainable. Of course the value of γ found using this method has an unknown level of theoretical uncertainty. However, it may be very useful in resolving ambiguities.

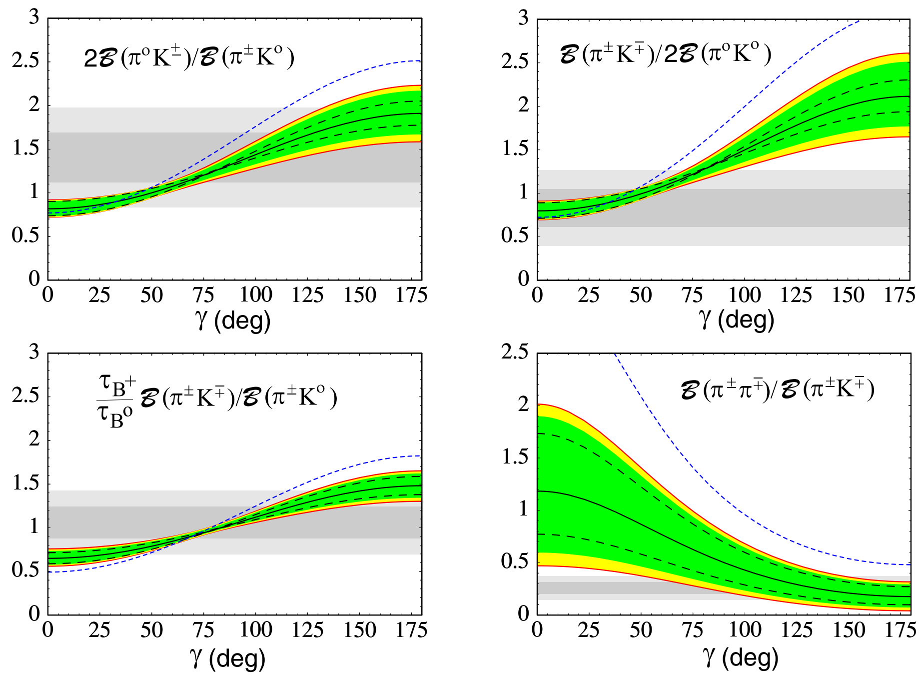

Figure 1.16: Model predictions from Beneke *et al.* as a function of the indicated rate ratios. The dotted curve shows the predictions from naive factorization. The curved bands show the total model uncertainties where the inner band comes from theoretical input uncertainties, while the outer band allows for errors to corrections on the theory. The specific sensitivity to $|V_{ub}|$ is showed as the dashed curves. The gray bands show the current data with a 1σ error while the lighter bands are at 2σ.

1.8.4 Measurement of γ Using CP Asymmetries in $B^o \to \pi^+\pi^-$ and $B_s^o \to K^+K^-$

Yet another interesting method for determining γ has been suggested by Fleischer [66]. The decays $B^o \to \pi^+\pi^-$ and $B_s^o \to K^+K^-$ are related to each other by interchanging all down and strange quarks, which is called U-spin flavor symmetry [67]. Both channels can occur via penguin or singly-Cabibbo suppressed tree levels diagrams, shown in Fig. 1.17.

For $B^o \to \pi^+\pi^-$ the transition amplitude is given by

$$A(B_d^0 \to \pi^+\pi^-) = \lambda_u^{(d)}\left(A_{\rm cc}^u + A_{\rm pen}^u\right) + \lambda_c^{(d)} A_{\rm pen}^c + \lambda_t^{(d)} A_{\rm pen}^t\,, \tag{1.58}$$

where $A_{\rm cc}^u$ is due to the tree contributions, and the amplitudes $A_{\rm pen}^j$ describe penguin topologies with internal j quarks ($j \in \{u, c, t\}$). These penguin amplitudes take into account both

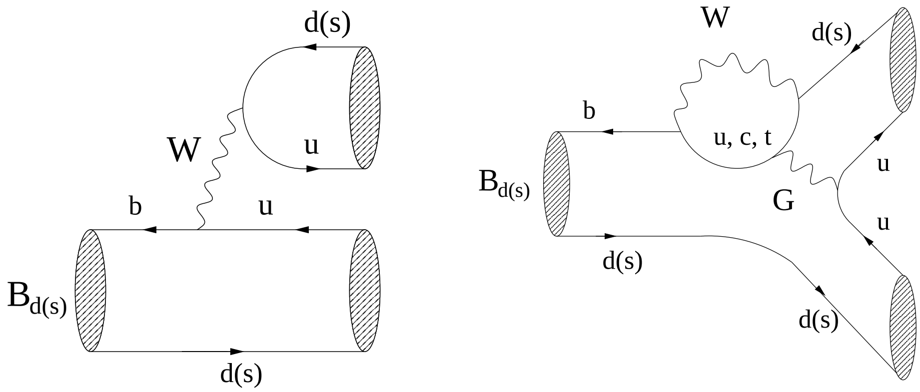

Figure 1.17: Feynman diagrams contributing to $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ (from Fleischer).

QCD and electroweak penguin contributions. The quantities

$$\lambda_j^{(d)} \equiv V_{jd}V_{jb}^* \tag{1.59}$$

are the usual CKM factors. If we make use of the unitarity of the CKM matrix and use the Wolfenstein parameterization, we have

$$A(B_d^0 \to \pi^+\pi^-) = e^{i\gamma}\left(1-\frac{\lambda^2}{2}\right)\mathcal{C}\left[1-d\,e^{i\theta}e^{-i\gamma}\right], \tag{1.60}$$

where

$$\mathcal{C} \equiv \lambda^3 A\, R_b\left(A_{\rm cc}^u + A_{\rm pen}^{ut}\right) \tag{1.61}$$

with $A_{\rm pen}^{ut} \equiv A_{\rm pen}^u - A_{\rm pen}^t$, and

$$d\,e^{i\theta} \equiv \frac{1}{(1-\lambda^2/2)R_b}\left(\frac{A_{\rm pen}^{ct}}{A_{\rm cc}^u + A_{\rm pen}^{ut}}\right). \tag{1.62}$$

The quantity $A_{\rm pen}^{ct}$ is defined in analogy to $A_{\rm pen}^{ut}$, and the CKM factors are given by

$$\lambda \equiv |V_{us}| = 0.22\,, \quad A \equiv \frac{1}{\lambda^2}\,|V_{cb}| \sim 0.8\,, \quad R_b \equiv \frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \sim 0.4. \tag{1.63}$$

For the following considerations, time-dependent CP asymmetries play a key role. In the case of a general B_d decay into a final CP eigenstate $|f\rangle$, satisfying

$$(\mathcal{CP})|f\rangle = \eta\,|f\rangle, \tag{1.64}$$

where η here is not the Wolfenstein parameter, we have (see equation 1.37)

$$\begin{aligned} a_{\rm CP}(B_d(t)\to f) &\equiv \frac{\Gamma(B_d^0(t)\to f) - \Gamma(\overline{B_d^0}(t)\to f)}{\Gamma(B_d^0(t)\to f) + \Gamma(\overline{B_d^0}(t)\to f)} \\ &= \mathcal{A}_{\rm CP}^{\rm dir}(B_d\to f)\cos(\Delta m_d t) + \mathcal{A}_{\rm CP}^{\rm mix}(B_d\to f)\sin(\Delta m_d t)\,. \end{aligned} \tag{1.65}$$

For the case of $B^o \to \pi^+\pi^-$, the decay amplitude takes the same form as (1.60), and we obtain the following expressions for the "direct" and "mixing-induced" CP-violating observables:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to f) \;=\; -\left[\frac{2\,d\sin\theta\sin\gamma}{1-2\,d\cos\theta\cos\gamma+d^2}\right] \tag{1.66}$$

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to f) \;=\; \eta\left[\frac{\sin(2\beta+2\gamma)-2\,d\,\cos\theta\,\sin(2\beta+\gamma)+d^2\sin 2\beta}{1-2\,d\cos\theta\cos\gamma+d^2}\right], \tag{1.67}$$

where η is equal to $+1$; for negligible values of the "penguin parameter" d, we have $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-) = \sin(2\beta+2\gamma) = -\sin(2\alpha)$. However, the penguin contributions are expected to play an important role.

Consider now the decay $B_s^0 \to K^+K^-$. It originates from $\bar{b} \to \bar{u}u\bar{s}$ quark-level processes, as can be seen in Fig. 1.17. Using a notation similar to that in (1.60), we obtain

$$A(B_s^0 \to K^+K^-) = e^{i\gamma}\lambda\,\mathcal{C}'\left[1+\left(\frac{1-\lambda^2}{\lambda^2}\right)d'e^{i\theta'}e^{-i\gamma}\right], \tag{1.68}$$

where

$$\mathcal{C}' \equiv \lambda^3 A\,R_b\left(A_{\rm cc}^{u'}+A_{\rm pen}^{ut'}\right) \tag{1.69}$$

and

$$d'e^{i\theta'} \equiv \frac{1}{(1-\lambda^2/2)R_b}\left(\frac{A_{\rm pen}^{ct'}}{A_{\rm cc}^{u'}+A_{\rm pen}^{ut'}}\right) \tag{1.70}$$

correspond to (1.61) and (1.62), respectively. The primes remind us that we are dealing with a $\bar{b} \to \bar{s}$ transition. It should be emphasized that (1.60) and (1.68) are completely general parameterizations of the $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ decay amplitudes within the Standard Model, relying only on the unitarity of the CKM matrix. In particular, these expressions take into account also final-state interaction effects, which can be considered as long-distance penguin topologies with internal up- and charm-quark exchanges.

There may be a sizeable width difference $\Delta\Gamma_s \equiv \Gamma_{\rm H}^{(s)} - \Gamma_{\rm L}^{(s)}$ between the B_s mass eigenstates [68], which may allow studies of CP violation with "untagged" B_s data samples [25]. Such untagged rates take the following form:

$$\Gamma(B_s^0(t) \to f) + \Gamma(\overline{B_s^0}(t) \to f) \propto R_{\rm H}\,e^{-\Gamma_{\rm H}^{(s)}t} + R_{\rm L}\,e^{-\Gamma_{\rm L}^{(s)}t}, \tag{1.71}$$

whereas the time-dependent CP asymmetry is given by

$$\begin{aligned} a_{\rm CP}(B_s(t) \to f) &\equiv \frac{\Gamma(B_s^0(t) \to f) - \Gamma(\overline{B_s^0}(t) \to f)}{\Gamma(B_s^0(t) \to f) + \Gamma(\overline{B_s^0}(t) \to f)} \\ &= 2\,e^{-\Gamma_s t}\left[\frac{\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to f)\cos(\Delta m_s t) + \mathcal{A}_{\rm CP}^{\rm mix}(B_s \to f)\sin(\Delta m_s t)}{e^{-\Gamma_{\rm H}^{(s)}t} + e^{-\Gamma_{\rm L}^{(s)}t} + \mathcal{A}_{\Delta\Gamma}(B_s \to f)\left(e^{-\Gamma_{\rm H}^{(s)}t} - e^{-\Gamma_{\rm L}^{(s)}t}\right)}\right] \end{aligned} \tag{1.72}$$

with $\mathcal{A}_{\Delta\Gamma}(B_s \to f) = (R_{\rm H} - R_{\rm L})/(R_{\rm H} + R_{\rm L})$. If the $B_s^0 \to f$ decay amplitude takes the same form as (1.68), we have

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to f) = + \left[\frac{2\,\tilde{d}' \sin\theta' \sin\gamma}{1 + 2\,\tilde{d}' \cos\theta' \cos\gamma + \tilde{d}'^2}\right] \tag{1.73}$$

$$\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to f) = + \eta \left[\frac{\sin(2\chi + 2\gamma) + 2\,\tilde{d}'\, \cos\theta'\, \sin(2\chi + \gamma) + \tilde{d}'^2 \sin 2\chi}{1 + 2\,\tilde{d}' \cos\theta' \cos\gamma + \tilde{d}'^2}\right] \tag{1.74}$$

$$\mathcal{A}_{\Delta\Gamma}(B_s \to f) = - \eta \left[\frac{\cos(2\chi + 2\gamma) + 2\,\tilde{d}' \cos\theta' \cos(2\chi + \gamma) + \tilde{d}'^2 \cos 2\chi}{1 + 2\,\tilde{d}' \cos\theta' \cos\gamma + \tilde{d}'^2}\right]. \tag{1.75}$$

These observables are not independent quantities, and satisfy the relation

$$\Big[\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to f)\Big]^2 + \Big[\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to f)\Big]^2 + \Big[\mathcal{A}_{\Delta\Gamma}(B_s \to f)\Big]^2 = 1. \tag{1.76}$$

In the general expressions (1.73)–(1.75), we have introduced the abbreviation

$$\tilde{d}' \equiv \left(\frac{1 - \lambda^2}{\lambda^2}\right) d', \tag{1.77}$$

and $2\chi = 2\arg(V_{ts}^* V_{tb})$ denotes the B_s^o–$\overline{B_s^o}$ mixing phase. Within the Standard Model, we have $2\chi \approx 0.03$ due to a Cabibbo suppression of $\mathcal{O}(\lambda^2)$, implying that 2χ is very small. This phase can be determined using $B_s \to J/\psi\, \eta\prime$ decays (see section 1.12.2).

Since the decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related to each other by interchanging all strange and down quarks, the U-spin flavor symmetry of strong interactions implies

$$d' = d \tag{1.78}$$
$$\theta' = \theta. \tag{1.79}$$

In contrast to certain isospin relations, electroweak penguins do not lead to any problems in the U-spin relations (1.78) and (1.79), according to Fleischer.

In general we have five physics quantities of interest, 2χ, d, θ, 2β and γ. Let us now assume that $\sin(2\beta)$ will be measured and $\sin(2\chi)$ either measured or tightly limited. Only d, θ and γ then need to be determined.

We have four possible measured quantities provided by the time-dependent CP asymmetries of the modes $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$. These four quantities are $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$, $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$, $\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)$ and $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$. To implement this plan we need measure only 3 of these four quantities, or combinations of them. For example, it may be difficult to independently determine $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$, because of the small number of observable B^o oscillations before the exponential decay reduces the number of events too much. However, the sum

$$\begin{aligned} a_{CP}^{\pi^+\pi^-} &= \int_0^\infty \mathcal{A}_{\rm CP}^{\rm dir} \cos(\Delta m_d t) + \mathcal{A}_{\rm CP}^{\rm mix} \sin(\Delta m_d t) \\ &= \frac{1}{1 + x^2}\mathcal{A}_{\rm CP}^{\rm dir} + \frac{x}{1 + x^2}\mathcal{A}_{\rm CP}^{\rm mix} \end{aligned} \tag{1.80}$$

can be determined and used with the other two measurements from $B_s^o \to K^+K^-$. Clearly other scenarios are possible.

1.8.5 Opportunities with B_s Mesons if $\Delta\Gamma$ is $\sim$10%

Measurement of $\Delta\Gamma$ can be used to estimate in an interesting but model dependent manner the value of Δm_s and thus provides a redundant check on B_s mixing measurements [25].

Should a large enough $\Delta\Gamma$ be determined there exist other possible ways to determine some of the interesting physics quantities discussed above. Some of these studies can be done without flavor tagging. In fact, the time evolution of untagged observables for a B_s decay into a vector-vector final state is proportional to

$$\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right) \sin\phi_{CKM}, \tag{1.81}$$

where ϕ_{CKM} is a CP violating angle from the CKM matrix and depends on the specific decay mode.

In general the angular distribution for $B_s \to VV$ is expressed in terms of transversity in a manner similar to equation 1.47, with the major difference being that the angular variables are time dependent. The time evolution of the decay $B_s \to J/\psi\phi$ is given in Table 1.7 [69].

Observable	Time evolution
$\lvert A_0(t)\rvert^2$	$\lvert A_0(0)\rvert^2 \left[e^{-\Gamma_L t} - e^{-\overline{\Gamma} t}\sin(\Delta m t)\sin(2\chi)\right]$
$\lvert A_\parallel(t)\rvert^2$	$\lvert A_\parallel(0)\rvert^2 \left[e^{-\Gamma_L t} - e^{-\overline{\Gamma} t}\sin(\Delta m t)\sin(2\chi)\right]$
$\lvert A_\perp(t)\rvert^2$	$\lvert A_\perp(0)\rvert^2 \left[e^{-\Gamma_H t} + e^{-\overline{\Gamma} t}\sin(\Delta m t)\sin(2\chi)\right]$
$\mathrm{Re}(A_0^*(t)A_\parallel(t))$	$\lvert A_0(0)\rvert\lvert A_\parallel(0)\rvert\cos(\delta_2-\delta_1)\left[e^{-\Gamma_L t} - e^{-\overline{\Gamma} t}\sin(\Delta m t)\sin(2\chi)\right]$
$\mathrm{Im}(A_\parallel^*(t)A_\perp(t))$	$\lvert A_\parallel(0)\rvert\lvert A_\perp(0)\rvert \left[e^{-\overline{\Gamma} t}\sin(\delta_1 - \Delta m t) + \frac{1}{2}\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right)\cos(\delta_1)\sin(2\chi)\right]$
$\mathrm{Im}(A_0^*(t)A_\perp(t))$	$\lvert A_0(0)\rvert\lvert A_\perp(0)\rvert \left[e^{-\overline{\Gamma} t}\sin(\delta_2 - \Delta m t) + \frac{1}{2}\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right)\cos(\delta_2)\sin(2\chi)\right]$

Table 1.7: Time evolution of the decay $B_s \to J/\psi(\to l^+l^-)\phi(\to K^+K^-)$ of an initially (i.e. at $t=0$) pure B_s meson. $\delta_{1,2}$ are strong phase shifts.

Combining with the decay of the $\overline{B}_s$ the time evolution of the untagged sample is given by

$$\frac{d^3\Gamma(J/\psi(\to l^+l^-)\phi(\to K^+K^-))}{d\cos\theta\; d\varphi\; d\cos\psi} \propto \frac{9}{16\pi}\Bigg[\; 2|A_0(0)|^2 e^{-\Gamma_L t}\cos^2\psi(1-\sin^2\theta\cos^2\varphi)$$

$$+\sin^2\psi\{|A_\parallel(0)|^2 e^{-\Gamma_L t}(1-\sin^2\theta\sin^2\varphi) + |A_\perp(0)|^2 e^{-\Gamma_H t}\sin^2\theta\}$$

$$+\frac{1}{\sqrt{2}}\sin 2\psi\left\{|A_0(0)||A_\parallel(0)|\cos(\delta_2-\delta_1)e^{-\Gamma_L t}\sin^2\theta\sin 2\varphi\right\}$$

$$+\left\{\frac{1}{\sqrt{2}}|A_0(0)||A_\perp(0)|\cos\delta_2\sin 2\psi\sin 2\theta\cos\varphi\right.$$

$$\left.-|A_\parallel(0)||A_\perp(0)|\cos\delta_1\sin^2\psi\sin 2\theta\sin\varphi\right\}\frac{1}{2}\left(e^{-\Gamma_H t}-e^{-\Gamma_L t}\right)\delta\phi\ \Bigg]\ . \qquad (1.82)$$

Thus a study of the time dependent angular distributions can lead to a measurement of $\sin(2\chi)$, especially if $\Delta\Gamma$ is determined before hand. It is also possible to integrate over two of the angles if statistics is limited. The distribution in J/ψ decay angle can be written as

$$\frac{d\Gamma(t)}{d\cos\theta}\propto(|A_0(t)|^2+|A_\parallel(t)|^2)\frac{3}{8}(1+\cos^2\theta)+|A_\perp(t)|^2\ \frac{3}{4}\sin^2\theta \qquad (1.83)$$

where the CP violating angle originates from the imaginary parts of the interference terms in the A's.

Other final states have been suggested that provide a measurement of γ using the above ideas. One particularly interesting set of decays is $B_s\to K^{*+}K^{*-}$ and $B_s\to K^{*o}\bar{K}^{*o}$ [70].

Finally, it is important to realize that determination of a non-zero $\Delta\Gamma$ allows the measurement of $Re\left(\frac{q}{p}\cdot\frac{\bar{A}}{A}\right)$, that in turn allows the removal of the ambiguities in the CKM angle of interest [25]. For the B_s decays mentioned here this could be γ or χ.

1.9 Summary of Crucial Measurements for CKM Physics

Table 1.8 lists the most important physics quantities and the decay modes that can be used to measure them.

Other modes which also may turn out to be useful include $B^o\to D^{*+}\pi^-$ and its charge-conjugate [71], which measures $\sin(-2\beta-\gamma)$ albeit with a small ≈1% predicted asymmetry,[6] and $B\to K\pi$ modes which can be used to find γ albeit with theoretical uncertainties. There are three alternative ways to measure γ, discussed in section 1.8, which serve both to remove ambiguities and perform checks. It will be much more difficult to find other modes to check α, however. One approach is to measure the CP asymmetry in $B^o\to\pi^+\pi^-$ and use theoretical models to estimate the effects of penguin pollution. Minimally, a great deal would be learned about the models. It also turns out that the third ambiguity in α can be removed by comparing the CP violating asymmetry in $\pi^+\pi^-$ with that found in $\rho\pi$ and using some mild theoretical assumptions [53]. After the three angles α, β and γ have been measured, we need to check if they add up to 180^o. A discrepancy here would be unexpected. To be sure, this check is not complete if ambiguities have not been removed. (Even if the angles sum to 180^o, new physics could hide.)

[6] To measure a CP asymmetry this way requires using equations 1.52, and extracting the strong phase, amplitude ratio, and a small asymmetry: a very difficult task.

Table 1.8: Required CKM Measurements for b's

Physics Quantity	Decay Mode
$\sin(2\alpha)$	$B^o \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^o$
$\cos(2\alpha)$	$B^o \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^o$
$\text{sign}(\sin(2\alpha))$	$B^o \rightarrow \rho\pi$ & $B^o \rightarrow \pi^+\pi^-$
$\sin(\gamma)$	$B_s \rightarrow D_s^{\pm}K^{\mp}$
$\sin(\gamma)$	$B^- \rightarrow \overline{D}^0 K^-$
$\sin(\gamma)$	$B^o \rightarrow \pi^+\pi^-$ & $B_s \rightarrow K^+K^-$
$\sin(2\chi)$	$B_s \rightarrow J/\psi\eta\prime$, $J/\psi\eta$
$\sin(2\beta)$	$B^o \rightarrow J/\psi K_s$
$\cos(2\beta)$	$B^o \rightarrow J/\psi K^o$, $K^o \rightarrow \pi\ell\nu$
$\cos(2\beta)$	$B^o \rightarrow J/\psi K^{*o}$ & $B_s \rightarrow J/\psi\phi$
x_s	$B_s \rightarrow D_s^+\pi^-$
$\Delta\Gamma$ for B_s	$B_s \rightarrow J/\psi\eta\prime$, $D_s^+\pi^-$, K^+K^-

We also want to measure as precisely as possible the side of the **bd** triangle (see Fig. 1.2) that requires a precise measurement of B_s mixing [72]. The other side is proportional to the magnitude of V_{ub}. This will no doubt be measured by e^+e^- b-factories and the precision will be limited by theoretical concerns if form-factors in the exclusive decays and q^2 distributions in the inclusive decays have been decisively measured. It is possible that measuring the rate $\Lambda_b \rightarrow p\ell\nu$ or the ratio of this rate to $\Lambda_b \rightarrow \Lambda_c\ell\nu$, could help determine the theoretical uncertainties since the form-factors are different.

1.10 Rare Decays as Probes beyond the Standard Model

Rare decays have loops in the decay diagrams which makes them sensitive to high mass gauge bosons and fermions. Thus, they are sensitive to new physics. However, it must be kept in mind that any new effect must be consistent with already measured phenomena such as B_d^o mixing and $b \rightarrow s\gamma$.

These processes are often called "penguin" processes, for unscientific reasons [73]. A Feynman loop diagram is shown in Fig. 1.18 that describes the transition of a b quark into a charged -1/3 s or d quark, which is effectively a neutral current transition. The dominant charged current decays change the b quark into a charged +2/3 quark, either c or u.

The intermediate quark inside the loop can be any charge +2/3 quark. The relative size of the different contributions arises from different quark masses and CKM elements. For $b \rightarrow s$, in terms of the Cabibbo angle (λ=0.22), we have for t:c:u - λ^2:λ^2:λ^4. The mass dependence favors the t loop, but the amplitude for c processes can be quite large ≈30%. Moreover, as pointed out by Bander, Silverman and Soni [74], interference can occur between

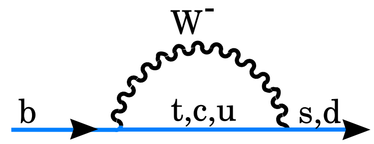

Figure 1.18: Loop or "Penguin" diagram for a $b \to s$ or $b \to d$ transition.

t, c and u diagrams and lead to CP violation. In the Standard Model it is not expected to occur when $b \to s$, due to the lack of a CKM phase difference, but could occur when $b \to d$. In any case, it is always worth looking for this effect; all that needs to be done, for example, is to compare the number of $K^{*-}\gamma$ events with the number of $K^{*+}\gamma$ events.

There are other possibilities for physics beyond the Standard Model to appear. For example, the W^- in the loop can be replaced by some other charged object such as a Higgs; it is also possible for a new object to replace the t.

1.10.1 $b \to s\gamma$

This process occurs when any of the charged particles in Fig. 1.18 emits a photon. CLEO first measured the inclusive rate [75] as well as the exclusive rate into $K^*(890)\gamma$ [76]. There is an updated CLEO measurement [77] using 1.5 times the original data sample and measurements from ALEPH [78] and BELLE [79].

To remove background CLEO used two techniques originally, one based on "event shapes" and the other on summing exclusively reconstructed B samples. CLEO uses eight different shape variables [75], and defines a variable r using a neural network to distinguish signal from background. The idea of the B reconstruction analysis is to find the inclusive branching ratio by summing over exclusive modes. The allowed hadronic system is composed of either a $K_S \to \pi^+\pi^-$ candidate or a $K^\mp$ combined with 1-4 pions, only one of which can be neutral. The restriction on the number and kind of pions maximizes efficiency while minimizing background. It does however lead to a model dependent error. Then both analysis techniques are combined. Currently, most of the statistical power of the analysis ($\sim$80%) comes from summing over the exclusive modes.

Fig. 1.19 shows the photon energy spectrum of the inclusive signal, compared with the model of Ali and Greub [80]. A fit to the model over the photon energy range from 2.1 to 2.7 GeV/c gives the branching ratio result shown in Table 1.9, where the first error is statistical and the second systematic.

ALEPH reduces the backgrounds by weighting candidate decay tracks in a $b \to s\gamma$ event by a combination of their momentum, impact parameter with respect to the main vertex and rapidity with respect to the b-hadron direction [78]. Their result is also listed in Table 1.9. The world average experimental value is also given, as well as the theoretical prediction.

The Standard Model prediction is in good agreement with the data. The consistency with Standard Model expectation has ruled out many models. Hewett has given a good review of the many minimal supergravity models which are excluded by the data [82]. Improved

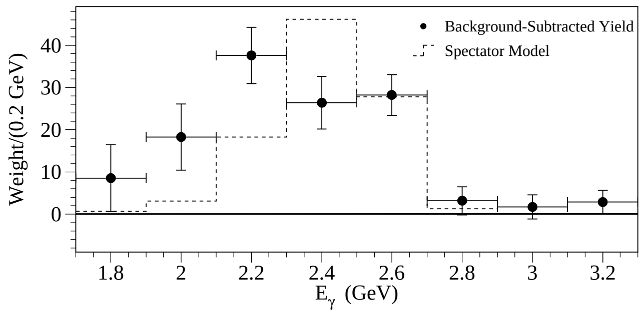

Figure 1.19: The background subtracted photon energy spectrum from CLEO. The dashed curve is a spectator model prediction from Ali and Greub.

Table 1.9: $\mathcal{B}(b \to s\gamma)$.

Experiment	$\mathcal{B} \times 10^{-4}$
CLEO	$3.21 \pm 0.43 \pm 0.27^{+0.18}_{-0.10}$
ALEPH	$3.11 \pm 0.80 \pm 0.72$
BELLE	$3.36 \pm 0.53 \pm 0.44^{+0.50}_{-0.54}$
Average	3.23 ± 0.42
Theory [81]	$(3.73 \pm 0.30) \times 10^{-4}$

experimental and theoretical accuracy are required to move beyond the Standard Model here. A measurement of $b \to d\gamma$ would be most interesting.

Triple gauge boson couplings are of great interest in checking the standard model. If there were an anomalous $WW\gamma$ coupling it would serve to change the Standard Model rate. $p\overline{p}$ collider experiments have also published results limiting such couplings [83]. In a two-dimensional space defined by $\Delta\kappa$ and λ, the D0 constraint appears as a tilted ellipse and the $b \to s\gamma$ as nearly vertical bands. In the standard model both parameters are zero.

1.10.2 The Exclusive Decays $K^*\gamma$ and $\rho\gamma$

The exclusive branching ratio is far more difficult to predict than the inclusive. CLEO measures $\mathcal{B}(B \to K^*(890)\gamma) = (4.2 \pm 0.8 \pm 0.6) \times 10^{-5}$, with this exclusive final state comprising $(18 \pm 7)\%$ of the total $b \to s\gamma$ rate [84]. BABAR [85] has made a more precise measurement separately for $K^{*o}\gamma$ of $(4.23 \pm 0.40 \pm 0.12) \times 10^{-5}$ and $K^{*+}\gamma$ of $(3.83 \pm 0.62 \pm 0.22) \times 10^{-5}$.

CLEO also limits $\mathcal{B}(B \to \rho\gamma) < 1.2 \times 10^{-5}$ at 90% confidence level [84]. This leads to a model dependent limit on $|V_{td}/V_{ts}|^2 < 0.45 - 0.56$, which is not very significant. It may be possible that improved measurements can find a meaningful limit, although that has been disputed [86].

1.10.3 $b \to s\ell^+\ell^-$

The diagrams that contribute to $b \to s\ell^+\ell^-$, where ℓ refers to either an electron or muon are shown in Fig. 1.20.

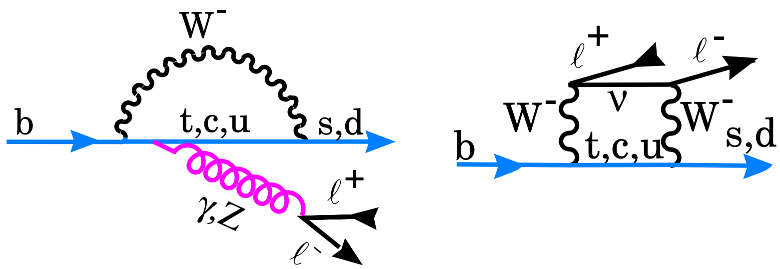

Figure 1.20: Loop or "Penguin" diagram for a $b \to s\ell^+\ell^-$ transition.

Since more diagrams contribute here than in $b \to s\gamma$, different physics can be probed. CP violation can be looked at in both the branching ratios and the polarization of the lepton pair [87]. When searching for such decays, care must be taken to eliminate the mass region in the vicinity of the J/ψ or ψ' resonances, lest these more prolific processes, that are not rare decays, contaminate the sample. Most searches have turned out negative. The results are listed in Table 1.10.

Table 1.10: Searches for $b \to s\ell^+\ell^-$ decays

b decay mode	90% c.l. upper limit	Group	Ali *et al.* Prediction [88]
$s\mu^+\mu^-$	50×10^{-6}	UA1 [89]	$(8 \pm 2) \times 10^{-6}$
	5.7×10^{-6}	CLEO [90]	
$K^{*o}\mu^+\mu^-$	4.0×10^{-6}	CDF [92]	2.9×10^{-6}
	23×10^{-6}	UA1 [89]	
	9.5×10^{-6}	CLEO [90]	
	3.2×10^{-6}	BABAR [91]	
$K^{*o}e^+e^-$	13×10^{-6}	CLEO [90]	5.6×10^{-6}
	6.6×10^{-6}	BABAR [91]	
$K^-\mu^+\mu^-$	9.7×10^{-6}	CLEO [90]	0.6×10^{-6}
	5.2×10^{-6}	CDF [92]	
	1.2×10^{-6}	BABAR [91]	
$K^-e^+e^-$	2.5×10^{-6}	CLEO [90]	0.6×10^{-6}
	0.8×10^{-6}	BABAR [91]	

The BELLE Collaboration [93] has claimed a positive signal for $\mathcal{B}(\overline{B}^o \to K^-\mu^+\mu^-) = (0.99^{+0.40+0.13}_{-0.32-0.14}) \times 10^{-6}$. Thus far this signal is has not been established in the analogous $K^-e^+e^-$ channel or by BABAR.

BTeV has the ability to search for both exclusive and inclusive dilepton final states. The inclusive measurement can be done following the techniques used by CLEO to discover inclusive $b \to s\gamma$ and set upper limits on $b \to s\ell^+\ell^-$. CLEO doesn't have vertex information, so they choose track combinations assigning a kaon hypothesis to one track and pion hypotheses to the other charged tracks. They allow up to four pions, only one of which can be neutral and proceed to reconstruct each combination as if it were an exclusive decay mode. If any combination succeeds, they keep it. BTeV can improve on this procedure in two ways. First of all BTeV will have RICH $K\pi$ separation. Secondly we can insist that the charged particles are consistent with coming from a b decay vertex. Of course, we lose the power of the beam energy constraint that is so efficient at rejecting background at the $\Upsilon(4S)$. However, it is a detailed question as to whether or not we more than make up the rejection power by using our advantages.

B's can also decay into dilepton final states. The Standard Model diagrams are shown in Fig. 1.21. In (a) the decay rate is proportional to $|V_{ub}|^2 f_B^2$. The diagram in (b) is much larger for B_s than B_d, again the factor of $|V_{ts}/V_{td}|^2$. Results of searches are given in Table 1.11.

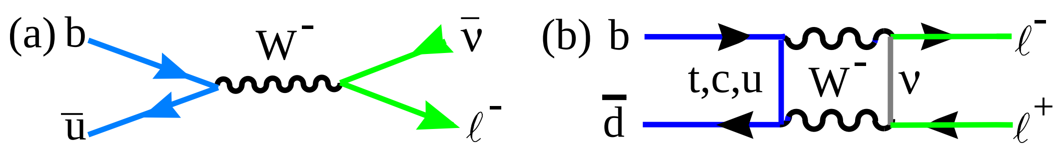

Figure 1.21: Decay diagrams resulting in dilepton final states. (a) is an annihilation diagram, and (b) is a box diagram.

Table 1.11: Upper limits on $b \to$ dilepton decays (@90% c.l.)

	$\mathcal{B}(B^o \to \ell^+\ell^-)$		$\mathcal{B}(B_s \to \ell^+\ell^-)$	$\mathcal{B}(B^- \to \ell^-\bar{\nu})$		
	e^+e^-	$\mu^+\mu^-$	$\mu^+\mu^-$	$e^-\bar{\nu}$	$\mu^-\bar{\nu}$	$\tau^-\bar{\nu}$
SM†	2×10^{-15}	8×10^{-11}	2×10^{-9}	10^{-15}	10^{-8}	10^{-5}
UA1 [89]		8.3×10^{-6}				
CLEO [94]	5.9×10^{-6}	5.9×10^{-6}		1.5×10^{-5}	2.1×10^{-5}	2.2×10^{-3}
CDF [95]		2.0×10^{-6}	6.8×10^{-6}			
ALEPH [96]						1.8×10^{-3}
L3 [97]						5.7×10^{-4}

†SM is the Standard Model prediction [98].

Searches for rare decays modes make up an important part of the BTeV physics program.

1.11 The Search for Mixing and CP Violation in Charm Decays

Predictions of the Standard Model contribution to mixing and CP violation in charm decay are small. Thus, this provides a good place to search for new physics.

The current experimental limit on charm mixing [99] is

$$r_D = \frac{1}{2}\left[\left(\frac{\Delta m_D}{\Gamma}\right)^2 + \left(\frac{\Delta\Gamma}{2\Gamma}\right)^2\right] < 5 \times 10^{-3} \quad , \tag{1.84}$$

while the Standard Model expectation is $\sim 10^{-6}$ [100] [101].

For CP violation the current limit is ~10% [12], while the Standard Model expectation is $\sim 10^{-3}$ [100] [102]. BTeV can probably reach the Standard Model level of CP violation in charm decays. (The D^{*+} provides a wonderful flavor tag.)

1.12 New Physics

1.12.1 Introduction

There are many reasons why we believe that the Standard Model is incomplete and there must be physics beyond. One is the plethora of "fundamental parameters," for example quark masses, mixing angles, etc... The Standard Model cannot explain the smallness of the weak scale compared to the GUT or Planck scales; this is often called "the hierarchy problem." It is believed that the CKM source of CP violation in the Standard Model is not large enough to explain the baryon asymmetry of the Universe [103]; we can also take the view that we will discover additional large unexpected effects in b and/or c decays. Finally, gravity is not incorporated. John Ellis said "My personal interest in CP violation is driven by the search for physics beyond the Standard Model" [104].

We must realize that *all* our current measurements are a combination of Standard Model and New Physics; any proposed models must satisfy current constraints. Since the Standard Model tree level diagrams are probably large, let's consider them a background to New Physics. Therefore loop diagrams and CP violation are the best places to see New Physics. The most important current constraints on New Physics models are

- The neutron electric dipole moment, $d_N < 6.3 \times 10^{-26}$e-cm.
- $\mathcal{B}(b \to s\gamma) = (3.23 \pm 0.42) \times 10^{-4}$ and $\mathcal{B}(b \to s\ell^+\ell^-) < 4.2 \times 10^{-5}$.
- CP violation in K_L decay, $\epsilon_K = (2.271 \pm 0.017) \times 10^{-3}$.
- B^o mixing parameter $\Delta m_d = (0.487 \pm 0.014)$ ps^{-1}.

1.12.2 Generic Tests for New Physics

We can look for New Physics either in the context of specific models or more generically, for deviations from the Standard Model expectation.

One example is to examine the rare decays $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ for branching ratios and polarizations. According to Greub et al. [105], "Especially the decay into K^*

yields a wealth of new information on the form of the new interactions since the Dalitz plot is sensitive to subtle interference effects."

Another important tactic is to test for inconsistencies in Standard Model predictions independent of specific non-standard models. Recall that the unitarity of the CKM matrix allows us to construct six relationships shown as triangles in the complex plane in Fig. 1.1.

All six of these triangles can be constructed knowing four and only four independent angles such as β, γ, χ or χ' (see equation 1.9) [106][8][107]. (We could substitute α for γ.) We know that β is large and γ is also likely to be large, while χ is estimated to be small $\approx$0.02, but measurable, while χ' is likely to be much smaller.

It has been pointed out by Silva and Wolfenstein [106] that measuring only angles may not be sufficient to detect new physics. For example, suppose there is new physics that arises in $B^o-\overline{B}^o$ mixing. Let us assign a phase θ to this new physics. If we then measure CP violation in $B^o \to J/\psi K_s$ and eliminate any penguin pollution problems in using $B^o \to \pi^+\pi^-$, then we actually measure $2\beta' = 2\beta + \theta$ and $2\alpha' = 2\alpha - \theta$. So while there is new physics, we miss it, because $2\beta' + 2\alpha' = 2\alpha + 2\beta$ and $\alpha' + \beta' + \gamma = 180°$.

1.12.2.1 A Critical Check Using χ

The angle χ (see equation 1.9) can be extracted by measuring the time dependent CP violating asymmetry in the reaction $B_s \to J/\psi\eta^{(\prime)}$, or if one's detector is incapable of quality photon detection, the $J/\psi\phi$ final state can be used. However, in this case there are two vector particles in the final state, making this a state of mixed CP, requiring a time-dependent angular analysis to extract χ, that requires large statistics.

Measurements of the magnitudes of CKM matrix elements all come with theoretical errors. Some of these are hard to estimate. The best measured magnitude is that of $\lambda = |V_{us}/V_{ud}| = 0.2205 \pm 0.0018$. Silva and Wolfenstein [106] [8] show that the Standard Model can be checked in a profound manner by seeing if:

$$\sin\chi = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{\sin\beta\ \sin\gamma}{\sin(\beta+\gamma)} \quad . \tag{1.85}$$

Here the precision of the check will be limited initially by the measurement of $\sin\chi$, not of λ. This check can reveal new physics, even if other measurements have not shown any anomalies. Other relationships to check include:

$$\sin\chi = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\sin\gamma\ \sin(\beta+\gamma)}{\sin\beta} \quad , \qquad \sin\chi = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\sin\beta\ \sin(\beta+\gamma)}{\sin\gamma} \quad . \tag{1.86}$$

These two equations lead to the non-trivial relationship:

$$\sin^2\beta \left|\frac{V_{td}}{V_{ts}}\right|^2 = \sin^2\gamma \left|\frac{V_{ub}}{V_{cb}}\right|^2 \quad . \tag{1.87}$$

This constrains these two magnitudes in terms of two of the angles. Note, that it is in principle possible to determine the magnitudes of $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$ without model dependent errors by measuring β, γ and χ accurately. Alternatively, β, γ and λ can be used to give a much more precise value than is possible at present with direct methods. For example, once β and γ are known $|V_{ub}/V_{cb}|^2 = \lambda^2 \sin^2\beta / \sin^2(\beta+\gamma)$.

1.12.2.2 Finding Inconsistencies

Another interesting way of viewing the physics was given by Peskin [108]. Non-Standard Model physics would show up as discrepancies among the values of (ρ, η) derived from independent determinations using CKM magnitudes ($|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$), or B_d^o mixing (β and α), or B_s mixing (χ and γ). Peskin destinguishes among four classes of CP violation measurements, corresponding to four different physical systems, such that each class would determine the unitarity triangle completely if the CKM model were a complete description of CP violation. This test of the CKM model comes from the comparison of the triangles shown in Figure 1.22, with error boxes for the sides or angles that might, he believes, be realized within the next decade.

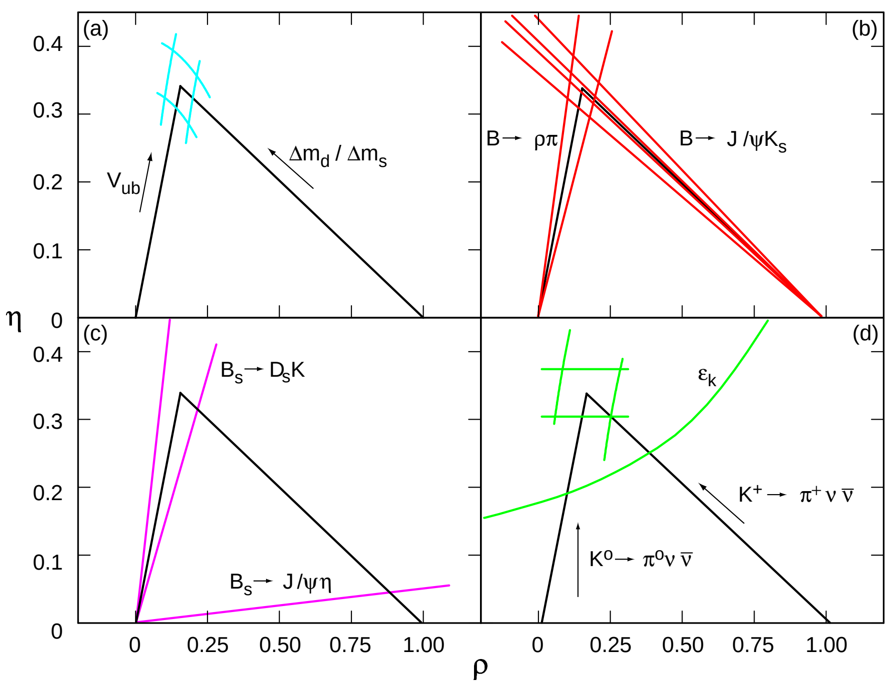

Figure 1.22: Illustration of four determinations of the unitarity triangle, by (a) non-CP observables, (b) B asymmetries, (c) B_s asymmetries, (d) K rare decays from Peskin [108].

Figure 1.22(a) shows the 'non-CP triangle'. This triangle takes advantage of the fact that one can determine the unitarity triangle by measuring the absolute values of CKM matrix elements and thus show the existence of the phase through non-CP-violating observables.

Figure 1.22(b) shows the 'B triangle'. This triangle is constructed from the CP asymmetries in $B^o/\overline{B}^o$ decays. To draw the figure, Peskin used the asymmetry in $B \to J/\psi K_S^o$

and the asymmetry in $B \to \rho\pi$. (Ignoring the discrete ambiguities in determining the CKM angles from the measured asymmetries.) Both of these asymmetries involve the phase in the B^o–$\overline{B}^o$ mixing amplitude and are sensitive to new physics through this source.

Figure 1.22(c) shows the 'B_s triangle'. The time-dependent CP asymmetry in $B_s \to D_s^{\pm}K^{\mp}$ is connected to $\sin\gamma$. The B_s system also allows an interesting null experiment. The time-dependent CP violation in $B_s \to c\bar{c}s\bar{s}$ decays is expected to be very small in the Standard Model. Thus the phase in $B_s \to J/\psi\eta$ will be a very sensitive indicator for new CP violating physics in the B_s–$\bar{B}_s$ mixing amplitude. This constraint is shown, just for the purpose of illustration, as a constraint on the base of the unitarity triangle.

Figure 1.22(d) shows the 'K triangle'. This is the triangle determined by two rare K decays $K^+ \to \pi^+\nu\bar{\nu}$, with a Standard Model amplitude approximately proportional to V_{td}, and $K_L^o \to \pi^o\nu\bar{\nu}$, a CP-violating process with a Standard Model amplitude proportional to $\mathrm{Im}[V_{td}]$. These decays proceed through box diagrams which could well have exotic contributions from new particles with masses of a few hundred GeV. Though Peskin says that: "The rare K decays are frighteningly difficult to detect."

1.12.3 New Physics Tests in Specific Models

1.12.3.1 Supersymmetry

Supersymmetry is a kind of super-model. The basic idea is that for every fundamental fermion there is a companion boson and for every boson there is a companion fermion. There are many different implementations of couplings in this framework [109]. In the most general case we pick up 80 new constants and 43 new phases. This is clearly too many to handle so we can try to see things in terms of simpler implementations. In the minimal model (MSSM) we have only two new fundamental phases. One, θ_D, would arise in B^o mixing and the other, θ_A, would appear in B^o decay. A combination would generate CP violation in D^o mixing, call it $\phi_{K\pi}$ when the $D^o \to K^-\pi^+$ [110]. Table 1.12 shows the CP asymmetry in three different processes in the Standard Model and the MSSM.

Table 1.12: CP Violating Asymmetries in the Standard Model and the MSSM.

Process	Standard Model	New Physics
$B^o \to J/\psi K_s$	$\sin 2\beta$	$\sin 2(\beta + \theta_D)$
$B^o \to \phi K_s$	$\sin 2\beta$	$\sin 2(\beta + \theta_D + \theta_A)$
$D^o \to K^-\pi^+$	0	$\sim \sin\phi_{K\pi}$

Two direct effects of New Physics are clear here. First of all, the difference in CP asymmetries between $B^o \to J/\psi K_s$ and $B^o \to \phi K_s$ would show the phase ϕ_A. Secondly, there would be finite CP violation in $D^o \to K^-\pi^+$ where none is expected in the Standard Model.

Manifestations of specific SUSY models lead to different patterns. Table 1.13 shows the expectations for some of these models in terms of these variables and the neutron electric dipole moment d_N; see [110] for details. Note, that "Approximate CP" has already been

Table 1.13: Some SUSY Predictions.

Model	$d_N \times 10^{-25}$	θ_D	θ_A	$\sin\phi_{K\pi}$
Standard Model	$\leq 10^{-6}$	0	0	0
Approx. Universality	$\geq 10^{-2}$	$\mathcal{O}(0.2)$	$\mathcal{O}(1)$	0
Alignment	$\geq 10^{-3}$	$\mathcal{O}(0.2)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$
Heavy squarks	$\sim 10^{-1}$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(10^{-2})$
Approx. CP	$\sim 10^{-1}$	-β	0	$\mathcal{O}(10^{-3})$

ruled out by the measurements of $\sin 2\beta$.

In the context of the MSSM there will be significant contributions to B_s mixing, and the CP asymmetry in the charged decay $B^{\mp} \to \phi K^{\mp}$. The contribution to B_s mixing significantly enhances the CP violating asymmetry in modes such as $B_s \to J/\psi\eta$. (Recall the CP asymmetry in this mode is proportional to $\sin 2\chi$ in the Standard Model.) The Standard Model and MSSM diagrams are shown in Fig. 1.23. The expected CP asymmetry in the MSSM is $\approx \sin\phi_\mu \cos\phi_A \sin(\Delta m_s t)$, which is approximately 10 times the expected value in the Standard Model [111].

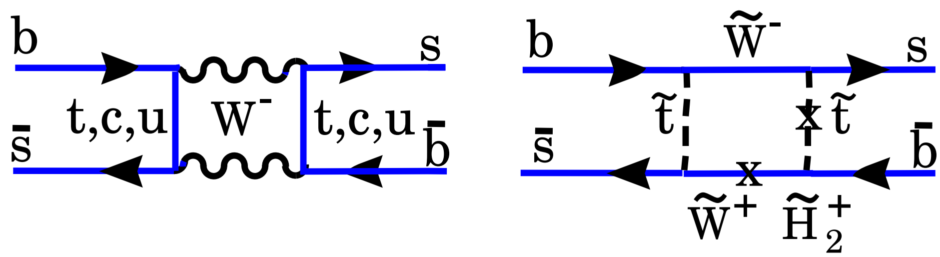

Figure 1.23: The Standard Model (left) and MSSM (right) contributions to B_s^o mixing.

We observed that a difference between CP asymmetries in $B^o \to J/\psi K_s$ and ϕK_s arises in the MSSM due to a CP asymmetry in the decay phase. It is possible to observe this directly by looking for a CP asymmetry in $B^{\mp} \to \phi K^{\mp}$. The Standard Model and MSSM diagrams are shown in Fig. 1.24. Here the interference of the two diagrams provides the CP asymmetry. The predicted asymmetry is equal to $(M_W/m_{squark})^2 \sin\phi_\mu$ in the MSSM, where m_{squark} is the relevant squark mass [111].

The ϕK and ϕK^* final states have been observed, first by CLEO [112] and subsequently by BABAR [113]. The average branching ratio is $\mathcal{B}(B^- \to \phi K^-) = (6.8 \pm 1.3) \times 10^{-6}$ showing that in principle large samples can be acquired especially at hadronic machines.

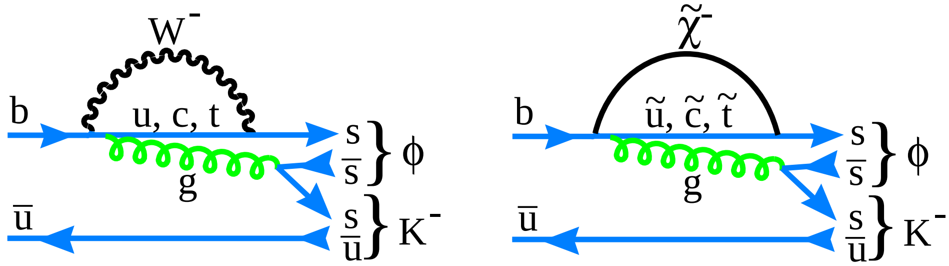

Figure 1.24: The Standard Model (left) and MSSM (right) contributions to $B^- \to \phi K^-$.

1.12.3.2 Other New Physics Models

There are many other specific models that predict New Physics in b decays. We list here a few of these with a woefully incomplete list of references, to give a flavor of what these models predict.

- *Two Higgs and Multi-Higgs Doublet Models*- They predict large effects in ϵ_K and CP violation in $D^o \to K^-\pi^+$ with only a few percent effect in B^o [110]. Expect to see 1-10% CP violating effects in $b \to s\gamma$ [114].

- *Left-Right Symmetric Model*- Contributions compete with or even dominate over Standard Model contributions to B_d and B_s mxing. This means that CP asymmetries into CP eigenstates could be substantially different from the Standard Model prediction [110].

- *Extra Down Singlet Quarks*- Dramatic deviations from Standard Model predictions for CP asymmetries in b decays are not unlikely [110].

- *FCNC Couplings of the Z boson*- Both the sign and magnitude of the decay leptons in $B \to K^*\ell^+\ell^-$ carry sensitive information on new physics. Potential effects are on the of 10% compared to an entirely negligable Standard Model asymmetry of $\sim 10^{-3}$ [115]. These models also predict a factor of 20 enhancement of $b \to d\ell^+\ell^-$ and could explain a low value of $\sin 2\beta$ [116].

- *Noncommutative Geometry*- If the geometry of space time is noncommutative, i.e. $[x_\mu, x_\nu] = i\theta_{\mu\nu}$, then CP violating effects may be manifest a low energy. For a scale <2 TeV there are comparable effects to the Standard Model [117].

- *MSSM without new flavor structure*- Can lead to CP violation in $b \to s\gamma$ of up to 5% [118]. Ali and London propose [119] that the Standard Model formulas are modified by Supersymmetry as

$$\Delta m_d = \Delta m_d(\mathrm{SM}) \left[1 + f\left(m_{\chi_2^\pm}, m_{\tilde{t}_R}, m_{H^\pm}, tan\beta\right)\right] \quad (1.88)$$

$$\Delta m_s = \Delta m_s(\mathrm{SM}) \left[1 + f\left(m_{\chi_2^\pm}, m_{\tilde{t}_R}, m_{H^\pm}, tan\beta\right)\right] \quad (1.89)$$

$$
\begin{aligned}
|\epsilon_K| &= \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} B_K (A^2\lambda^6\overline{\eta}) \left[y_c \left(\eta_{ct} f_3(y_c, y_t) - \eta_{cc}\right)\right. \\
& \left. +\eta_{tt} y_t f_s(y_t) \left[1 + f\left(m_{\chi_2^\pm}, m_{\tilde{t}_R}, m_{H^\pm}, tan\beta\right)\right] A^2\lambda^4(1-\overline{\rho})\right] \quad , \qquad (1.90)
\end{aligned}
$$

where $\Delta m(SM)$ refers to the Standard Model formula and the expression for $|\epsilon_K|$ would be the Standard Model expression if f were set equal to zero. Ali and London show that it is reasonable to expect that $0.8 > f > 0.2$, so since the CP violating angles will not change from the Standard Model, determining the value of $(\rho,\ \eta)$ using the magnitudes $\Delta m_s/\Delta m_d$ and $|\epsilon_K|$ will show an inconsistency with values obtained using other magnitudes and angles.

- *Extra Dimensions*- We are beginning to see papers predicting b decay phenomena when the world has extra dimensions. See [120].

We close this section with a quote from Masiero and Vives [121]: "The relevance of SUSY searches in rare processes is not confined to the usually quoted possibility that indirect searches can arrive 'first' in signaling the presence of SUSY. Even after the possible direct observation of SUSY particles, the importance of FCNC and CP violation in testing SUSY remains of utmost relevance. They are and will be complementary to the Tevatron and LHC establishing low energy supersymmetry as the response to the electroweak breaking puzzle."

We agree, except that we would replace "SUSY" with "New Physics." It is clear that precision studies of b decays can bring a wealth of information to bear on new physics, that probably will be crucial in sorting out anything seen at the LHC.

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